Exam II will test on Chapter 11 from the text. This exam will primarily focus on Fourier series representations of periodic functions. Specifically, of the five questions three will test your ability to calculate Fourier series. One question will be conceptual and focus on your understanding of the meaning of Fourier series and to some extent Fourier integrals and transforms. The final question will test your ability to calculate with Fourier transforms. To prepare for the exam you should refer to the homework assignments. The following is a list of concepts and methods which you should be familiar with. Also listed are the key equations from Chapter 11. It is assumed that the student has equations (1)-(4) and (9) memorized for the exam.

11.1-2 Fourier Series of Periodic Functions (Formulas (1)-(2))

From this section the student should understand:

- The concept of an orthogonal trigonometric system.
- The concept of representing periodic functions using trigonometric series.

From this section the student should be able to:

- Determine Fourier coefficients of a 2L-periodic function.
- Using the Fourier coefficients, write down the Fourier series representation of a 2L-periodic function.

11.3 Even and Odd Functions. Half-Range Expansions

From this section the student should understand:

- The algebraic and geometric properties of even and odd functions.
- The definite integral simplifications associated with even and odd functions.
- The Fourier series representations of functions with symmetry.

From this section the student should be able to:

- Simplify integrals based on symmetry of the integrand.
- Simplify Fourier series based on symmetry of the periodic function.
- Periodically extend functions whose domain is finite to get Half-Range series expansions.

11.4 Complex Fourier Series (Formulas (3)-(4))

From this section the student should understand:

- The connection between exponential functions and sine/cosine functions.
- The equivalence of the real Fourier series and the complex Fourier series.

From this section the student should be able to:

- Determine the complex Fourier coefficients of a 2L-periodic function.
- Using the complex Fourier coefficients, write down the complex Fourier series representation of a 2L-periodic function.
- Determine the equivalent real Fourier series representation of a function given the complex Fourier series.

11.7 Fourier Integral (Formulas (5)-(6))

From this section the student should understand:

- The connection between periodic functions and non-periodic functions.
- The relationship between the Fourier Series and the Fourier Integral.
- Simplifications of the Fourier Integral associated with non-periodic symmetric functions.

From this section the student should be able to:

- Determine the Fourier Integral representation of a non-periodic function.

11.8 Fourier Sine and Cosine Transforms (Formulas (7)-(8))

From this section the student should understand:

- The connection between the Fourier Integral representation of symmetric functions and the Fourier Sine/Cosine Transforms (both forward and inverse transforms).
- The connection between sine and cosine transforms and odd and even functions.

From this section the student should be able to:

- Given a function, find the sine/cosine transform.

11.9 The Fourier Transform (Formula (9))

From this section the student should understand:

- The connection between the Fourier Integral and the Fourier Transform.
- The concept of transform pairs.

From this section the student should be able to:

- Determine the Fourier Transform of a function.
- Determine the Fourier Transform of a function using transform pairs.

Important formulas: The following equations will be needed for the examination.

(1) $$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{L}x\right) + b_n \sin\left(\frac{n\pi}{L}x\right)$$

(2) $$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)dx, \qquad a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi}{L}x\right)dx, \qquad b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi}{L}x\right)dx.$$

(3) $$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{\frac{in\pi}{L}x}$$

(4) $$c_n = \frac{1}{2L}\int_{-L}^{L} f(x)e^{-\frac{in\pi}{L}x}dx$$

(5) $$f(x) = \int_0^{\infty} [A(\omega)\cos(\omega x) + B(\omega)\sin(\omega x)]d\omega$$

(6) $$A(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\cos(\omega x)dx, \qquad B(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\sin(\omega x)dx$$

(7) $$\hat{f}_c(\omega) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\cos(\omega x)dx, \qquad f(x) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} \hat{f}_c(\omega)\cos(\omega x)d\omega$$

(8) $$\hat{f}_s(\omega) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\sin(\omega x)dx, \qquad f(x) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} \hat{f}_s(\omega)\sin(\omega x)d\omega$$

(9) $$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-i\omega x}dx \qquad f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \hat{f}(\omega)e^{i\omega x}d\omega$$