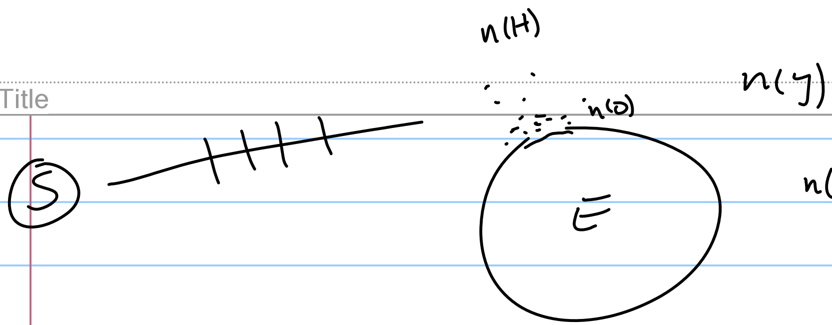
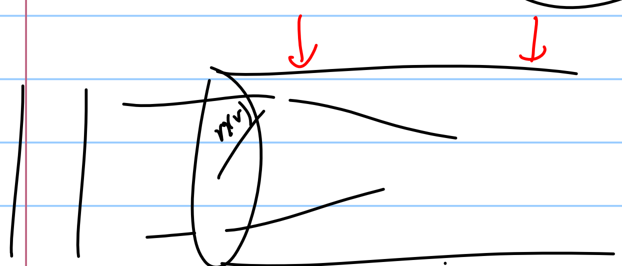
Note Title 7/12/2006



$n(H) < n(O)$



$n(0=r) > n(r=R)$

gradient index rod

Memorize ME in vac for exam.

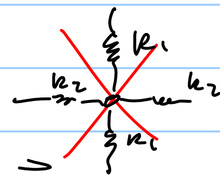
EM waves in dielectrics: interfaces

$$\vec{\nabla} \cdot \vec{D} = 0 \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{H} = -\frac{\partial \vec{D}}{\partial t}$$

Region of no free charge or current $\vec J = 0$

Linear medium $\quad \vec D = \epsilon \vec E \quad \vec H = \frac{\vec B}{\mu}$ & homogeneous



$$\vec D = \epsilon_0 \vec E + \underset{\epsilon_0 \chi_e \vec E}{\underset{\shortparallel}{\vec P}} = \epsilon \vec E \qquad \cancel{\vec P = \overleftrightarrow{\alpha}\, \vec E} \qquad c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$\frac{\partial^2 \vec E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \vec E}{\partial t^2}$$

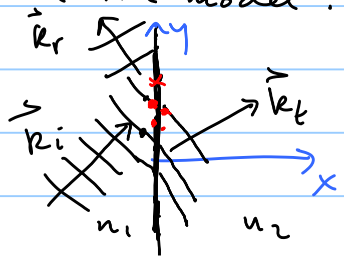
$$\left.\begin{aligned} \vec\nabla \cdot \vec E &= 0 \qquad & \vec\nabla \times \vec E &= -\frac{\partial \vec B}{\partial t} \\ \vec\nabla \cdot \vec B &= 0 \qquad & \vec\nabla \times \vec B &= \mu\epsilon \frac{\partial \vec E}{\partial t} \end{aligned}\right\}$$

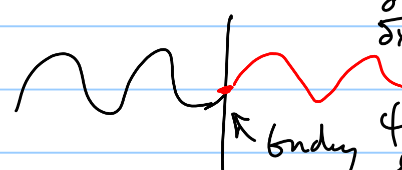
PDE wave eqn

$$v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{c}{n} = \frac{1}{\sqrt{\epsilon_0\mu_0}} \frac{\sqrt{\epsilon_0\mu_0}}{\sqrt{\epsilon\mu}}$$

$$n = \sqrt{\frac{\epsilon\mu}{\epsilon_0\mu_0}}$$

Do we have to know anything about microscopic physics for the model? No discussion of atoms.





$$\left.\frac{\partial \psi_l}{\partial x}\right|_{x=0} = \left.\frac{\partial \psi_r}{\partial x}\right|_{x=0}$$

bndry $\psi_{left} = \psi_{right}$

$$\boxed{\vec{\tilde{E}}_{0i}\, e^{i(\vec{k}_i\cdot\vec{r}-\cancel{\omega t})} + \vec{\tilde{E}}_{0r}\, e^{i(\vec{k}_r\cdot\vec{r}-\cancel{\omega t})} = \vec{\tilde{E}}_{0t}\, e^{i(\vec{k}_t\cdot\vec{r}-\cancel{\omega t})}}$$

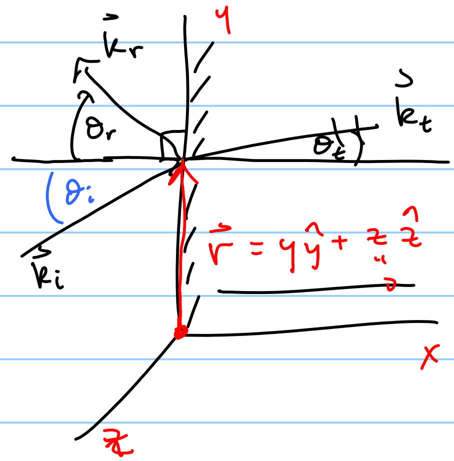
at bndry

at bndry $\vec{k}_i\cdot\vec{r}$, $\vec{k}_r\cdot\vec{r}$, $\vec{k}_t\cdot\vec{r}$ phase on bndry depends y & z

$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$
0 at bndy

$$\vec{k}_i\cdot\vec{r} = \vec{k}_r\cdot\vec{r} = \vec{k}_t\cdot\vec{r} \quad \text{for each point on interface}$$

Then each $e^{i\vec{k}_i\cdot\vec{r}} = e^{i\vec{k}_r\cdot\vec{r}} = e^{i\vec{k}_t\cdot\vec{r}}$ cancels in the above eqns.



$$\boxed{\vec{k}_i\cdot\vec{r} = \vec{k}_r\cdot\vec{r}} = |\vec{k}_r||\vec{r}|\cos\varphi$$

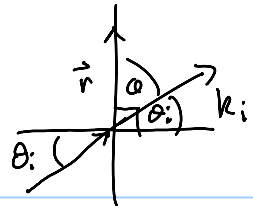
$$\boxed{\vec{k}_i\cdot\vec{r} = \vec{k}_t\cdot\vec{r}}$$

$$|\vec{k}_i| \stackrel{?}{=} |\vec{k}_r|$$

$$\frac{2\pi}{\lambda} = \frac{2\pi}{\lambda}$$

$$\vec{k}_i \cdot \vec{r} = |k_i||\vec{r}|\cos\varphi$$

$$= |k_i||\vec{r}|\sin\theta_i$$



$\cos\varphi = \sin\theta_i$

$$\vec{k}_t \cdot \vec{r} = |k_t||\vec{r}|\sin\theta_t$$

$|k_t| \neq |k_i|$

$$|k_i|\cancel{|\vec{r}|}\sin\theta_i = |k_t|\cancel{|\vec{r}|}\sin\theta_t$$

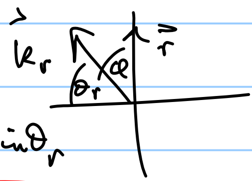
$$\frac{2\pi}{\frac{\lambda_{vac}}{n_i}}\sin\theta_i = \frac{2\pi}{\lambda_i}\sin\theta_i = \frac{2\pi}{\lambda_t}\sin\theta_t = \frac{2\pi}{\frac{\lambda_{vac}}{n_t}}\sin\theta_t$$

$$\frac{n_i\sin\theta_i}{\lambda_{vac}} = \frac{n_t\sin\theta_t}{\lambda_{vac}}$$

$$\boxed{n_i\sin\theta_i = n_t\sin\theta_t}$$

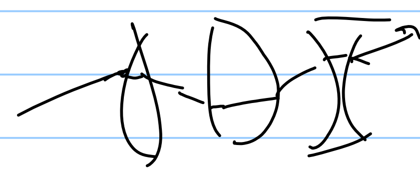
SNELL's law

$$\vec{k}_i \cdot \vec{r} = \vec{k}_r \cdot \vec{r}$$



$$\cancel{|k_i|}\,\cancel{|\vec{r}|}\sin\theta_i = \cancel{|k_r|}\,\cancel{|\vec{r}|}\sin\theta_r$$

$$\boxed{\sin\theta_i = \sin\theta_r}$$



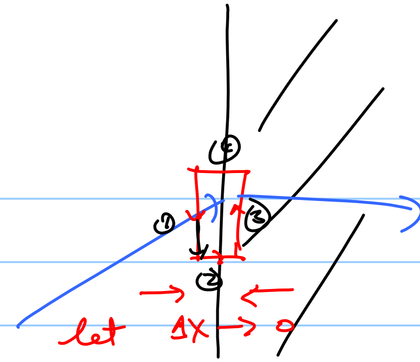
Bndry conditions:

(1) Faraday's Law $\Rightarrow$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Stokes $\downarrow$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{a}$$

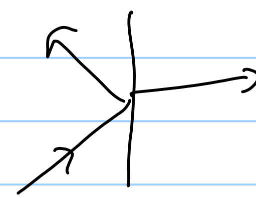


$$\int_{①} |\vec{E}||d\vec{l}|\cos\theta + \underbrace{\int_{②}^{\Delta x}}_{0\ \ \Delta x\to 0} + \int_{③} + \underbrace{\int_{④}^{\Delta x}}_{0\ \ \Delta x\to 0} = \underbrace{-\frac{\partial}{\partial t}\int B\cdot dy\,dx}_{\substack{B\ \text{area} = 0 \\ \| \\ L\,\Delta x \to 0}} = 0$$

$$|\vec{E}|\cos\theta = E_{\parallel}$$

$$\int_{①} + \int_{②} = E_{\parallel,1}L - E_{\parallel,2}L = 0$$

$$E_{\parallel,1} = E_{\parallel,2}$$

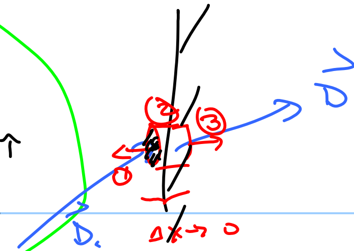


$$\boxed{\tilde{E}_{\parallel,i} + \tilde{E}_{\parallel,r} = \tilde{E}_{\parallel,t}}$$ Bndry condition

(2) Gauss's Law

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$
$$\downarrow$$
$$\oint \vec{E}\cdot d\vec{a} = \frac{\int \rho\, d\tau}{\epsilon_0}$$

before



$\rho_f = 0$ at interface

$$\vec{\nabla}\cdot\underset{\epsilon \vec{E}}{\underset{\shortparallel}{\vec{D}}} = \rho_f = 0$$

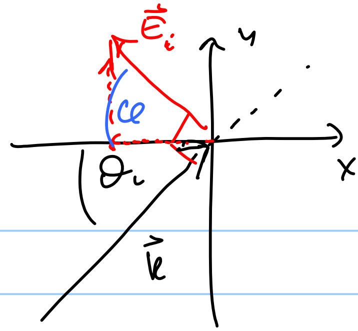
$$\int \underset{\shortparallel}{\vec{\nabla}\cdot\vec{D}\, d\tau} = \int 0\cdot d\tau = 0$$

$$\oint \underset{\epsilon E}{\underset{\shortparallel}{\vec{D}}}\cdot d\vec{a} = 0 \qquad \vec{E}\cdot d\vec{a} = |\vec{E}||d\vec{a}|\cos\theta$$

$$\underset{(1)}{\int} \vec{D}\cdot d\vec{a} + \underset{(2)}{\int} |\vec{D}||\underset{\cancel{dx\,dy}}{\underset{\shortparallel}{d\vec{a}}}|\cos\theta + \underset{(3)}{\int}$$

$$-D_{\perp,i}\,A + D_{\perp,t}\,A + \underset{0}{\angle} = 0 \qquad \epsilon_\ell E_{\perp,\ell} = D_{\perp,\ell} = D_{\perp,t} = \epsilon_t E_{\perp,t}$$

$$\boxed{\epsilon_\ell \tilde{E}_{\perp,i} + \epsilon_\ell \tilde{E}_{\perp,r} = \epsilon_t \tilde{E}_{\perp,t}}$$



E be in the y-z plane

$$E_{\perp,i} = |\vec{E}_i| \cos\varphi = |\vec{E}_i| \sin\theta_i$$

$\varphi = (90-\theta_i)$

neg x direction

Bndry condition

$$\epsilon_1 \left[-\tilde{E}_{0i} \sin\theta_i \, e^{i(\vec{k}_i\cdot\vec{r}-\omega t)} + \hat{\tilde{E}}_{0r} \sin\theta_r \, e^{i(k_r\cdot\vec{r}-\omega t)} \right]$$

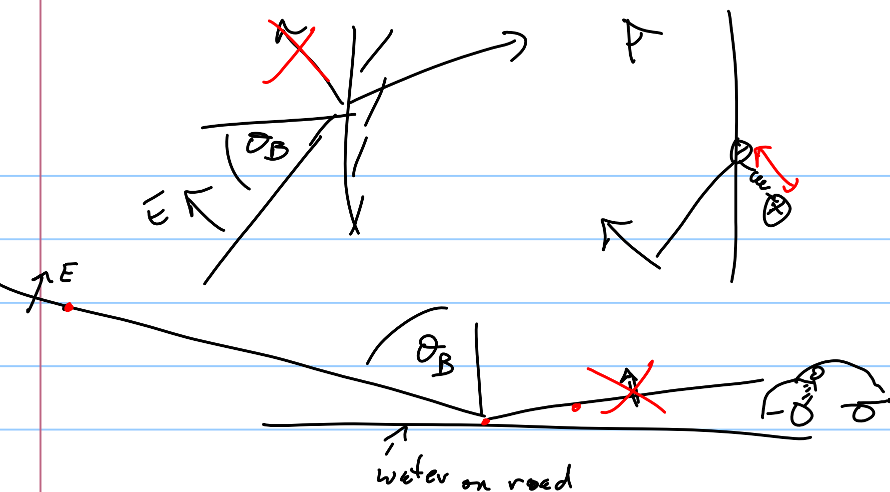
$$= \epsilon_2 \left(-\hat{\tilde{E}}_{0t} \sin\theta_t \, e^{i(\vec{k}_t\cdot\vec{r}-\omega t)} \right)$$

to satisfy this eqn we have $\vec{k}_i\cdot\vec{r} = \vec{k}_r\cdot\vec{r} = \vec{k}_t\cdot\vec{r}$

Leads to Fresnel eqns. Griffiths & Fowles

$$R = \frac{I_R}{I_i} \qquad T = \frac{I_T}{I_i}$$

(5)



EM waves in conductors

(1) Fields inside conductors only

(2) Reflection from a conducting surface

(1) M.E.

(a) $\vec{\nabla}\cdot\vec{E} = 0$ (c) $\vec{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t}$

(b) $\vec{\nabla}\cdot\vec{B} = 0$ (d) $\vec{\nabla}\times\vec{B} = \mu\epsilon\dfrac{\partial\vec{E}}{\partial t} + \mu\underset{\sigma E}{\vec{J}}$ ← Ohm's law

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} \vec{\nabla} \times \vec{B} = -\frac{\partial}{\partial t}\left(\mu\epsilon \frac{\partial E}{\partial t}\right) - \frac{\partial}{\partial t} \sigma E \mu$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2 \vec{E} = -\mu\epsilon \frac{\partial^2 \vec{E}}{\partial t} - \mu\sigma \frac{\partial \vec{E}}{\partial t}$$

$\vec{\nabla}\cdot\vec{E} = 0$ by (a)

$$\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} + \mu\sigma \frac{\partial \vec{E}}{\partial t}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} + \gamma \frac{\partial \psi}{\partial t}$$

Damping