Interaction of light with atoms: Absorption and gain

Connect propagation of beam to cross-sections

Examples for both absorption (pumping) and gain (amplifier)

Wave propagation with absorption

- Consider light absorption from a thin slab

$$I_1 = I_0 - I_0\,\alpha\,\Delta z$$

- Generalize to an equation for arbitrary length:

$$I_1 - I_0 = \Delta I = -I_0\,\alpha\,\Delta z \rightarrow \frac{dI}{dz} = -\alpha\,I$$

$$I(z) = I_0\,e^{-\alpha z}$$ Beer's Law

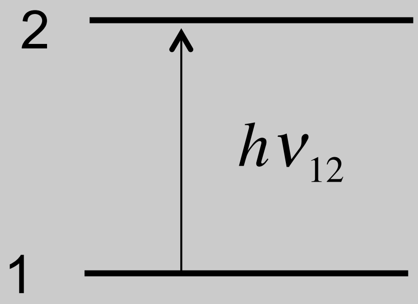
- Absorption coefficient (units m^{-1}) is proportional to the number density of absorbers:

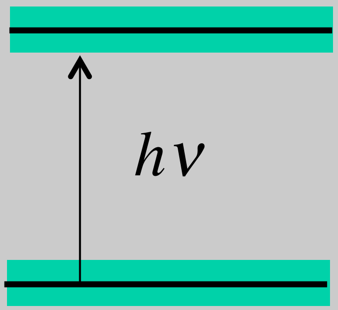
$$\alpha = N_1\sigma$$

  - N_1 = number density (m^{-3}) of species in level 1
  - σ? Has units of m^2, = "cross-section"

Models for σ: hard and soft spheres

- Consider an collection of “black” spheres that absorb if struck by a photon.
- Cross-section for absorption is just the projected area of the sphere. $\sigma = \pi a^2$
- For an atom, the probability of absorption depends on how close the incident frequency is to resonance:





Absorption lines are *broadened*, so *exact* energy is not required.

$$\sigma \rightarrow \sigma(\nu)$$

Example: absorption of pump light in Nd:YAG

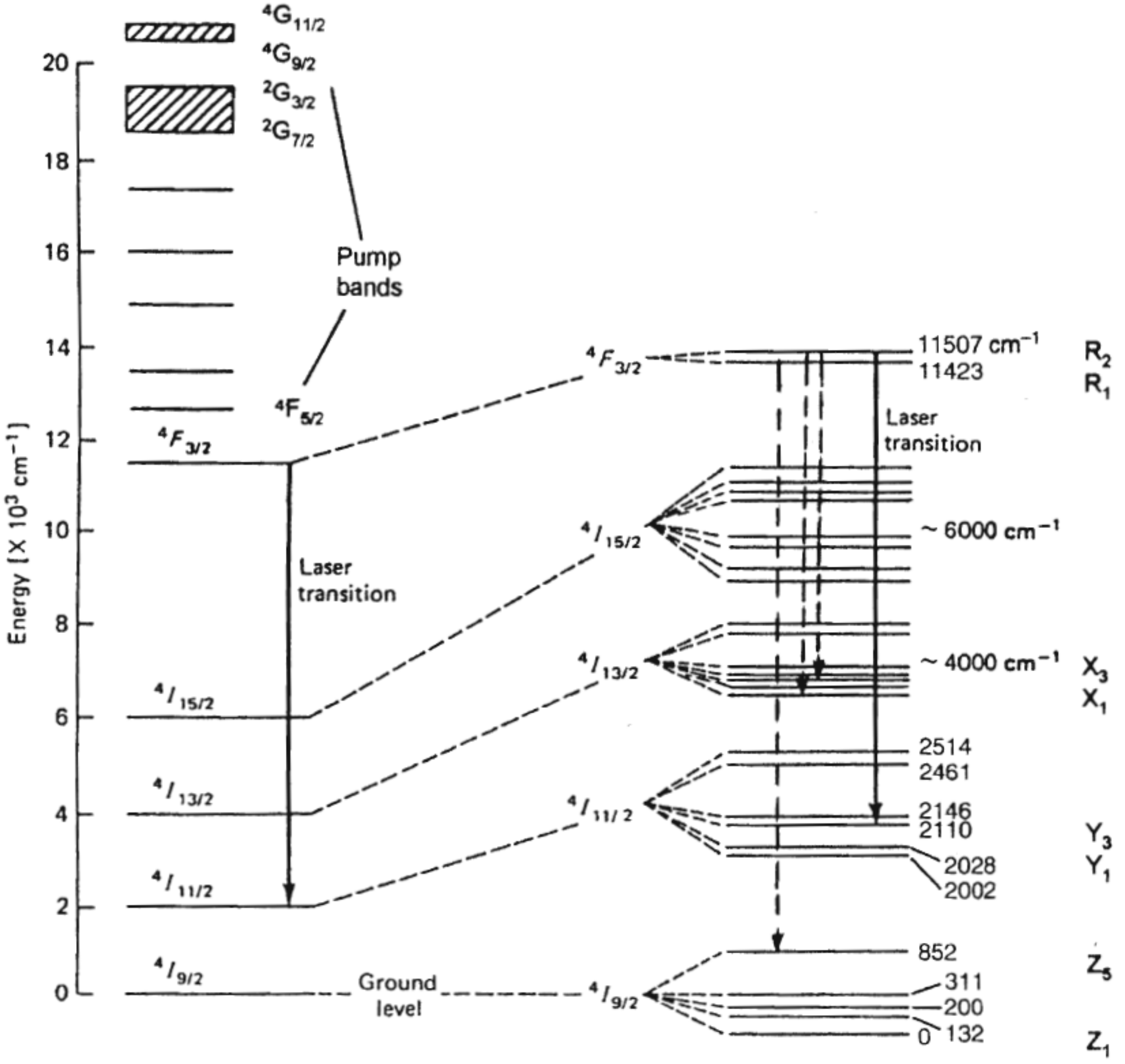
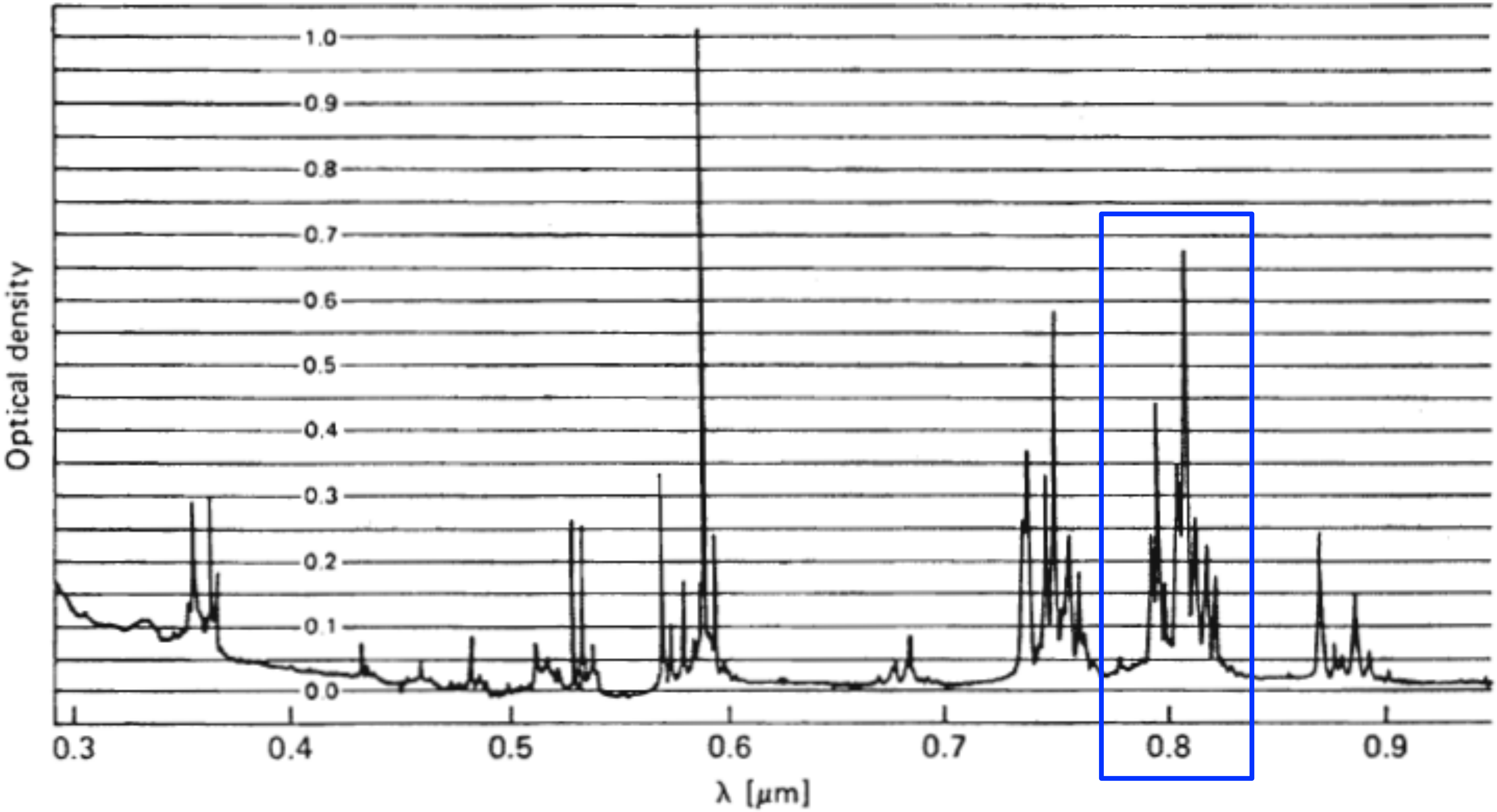


Fig. 2.2. Energy level diagram of Nd:YAG. The solid line represents the major transition at 1064 nm, and the dashed lines are the transitions at 1319, 1338, and 946 nm.

- Nd^{3+} is a heavy ion with many possible transitions
- Pump to anywhere above the $^4F_{3/2}$ level

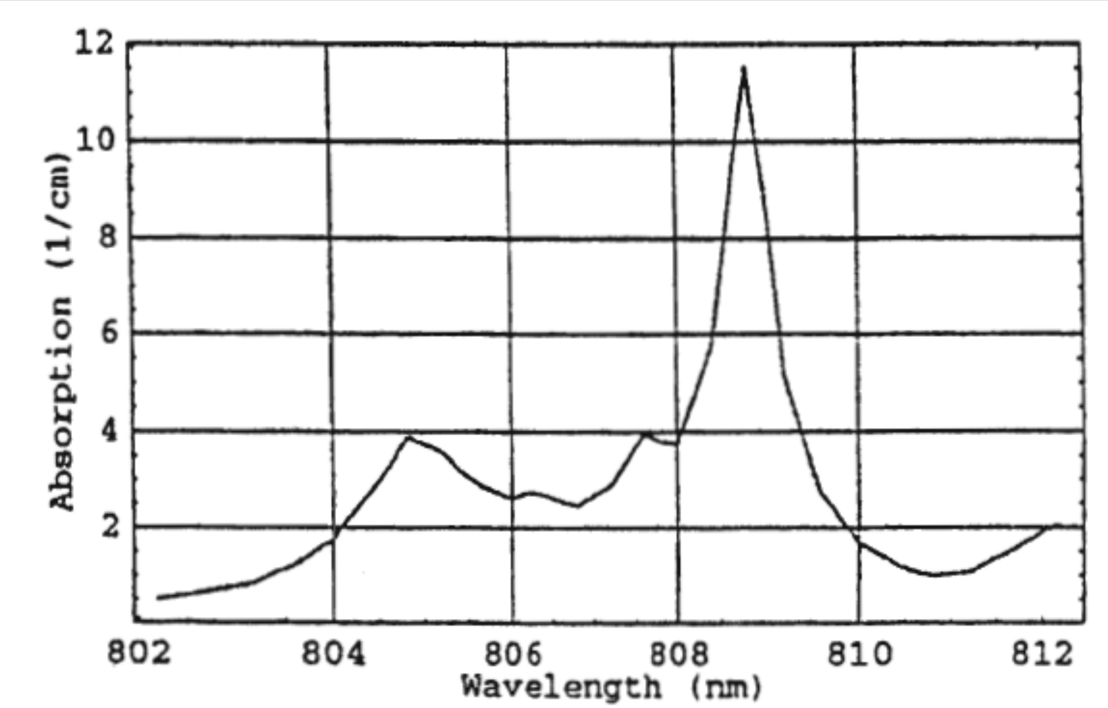
Absorption spectrum of Nd^{3+}:YAG



- Optical density (OD) = $-\log_{10}[T]$

Pump bands near 808nm

- Powerful laser diodes (LD) are available near 808nm



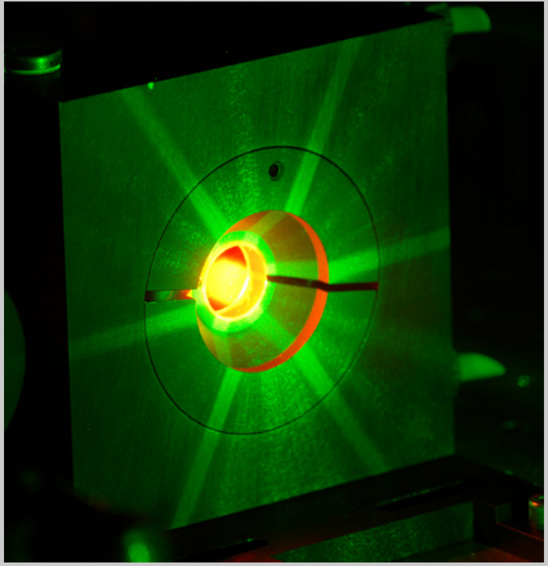
3mm thick Nd:YAG crystal

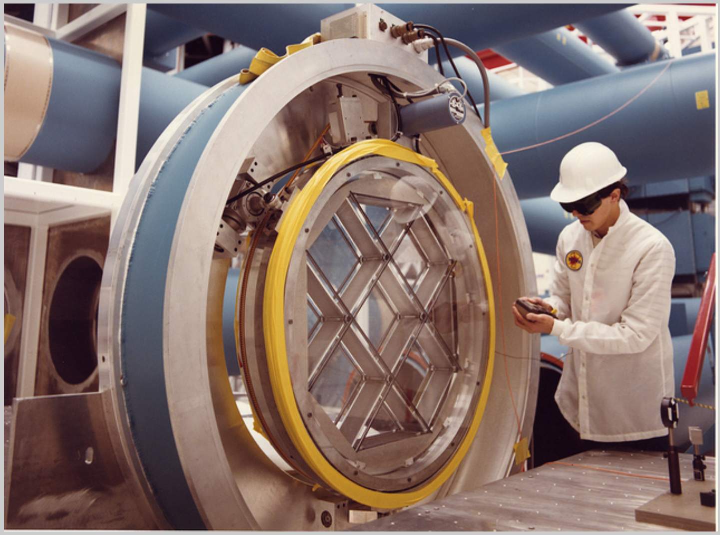
- What % is absorbed at the peak (α=11/cm)?
- What is the OD?
- If $N_{Nd}=1.38\times10^{20}/cm^3$ (1% atomic), what is the absorption cross-section?

- Note: LD output wavelength depends on temperature, so this needs tuning and stabilization in real systems.

Amplifiers: pumping and small-signal gain

- Absorption $I[z] = I_0 \exp[-N_0 \sigma_{12} z] = I_0 \exp[-\alpha z]$
- Gain $I[z] = I_0 \exp[N_{inv} \sigma_{21} z] = I_0 \exp[g z]$
  - What is the inversion density?
  - How to express it in terms of the pump distribution
  - How does gain depend on λ or ω ?
  - What happens when the inversion density is depleted?





Simple gain calculation

- Assume spatially uniform pump distribution

$G_0 = \exp\left[N_{inv}\sigma_{21} L\right]$ Small-signal gain

- Available energy for extraction:

$$E_{stor} = N_{inv} A L h\nu_{21} \rightarrow N_{inv} = \frac{E_{stor}}{A L h\nu_{21}}$$

A = area of beam

$$G_0 = \exp\left[\frac{E_{stor}}{A}\frac{\sigma_{21}}{h\nu_{21}}\right]$$

- Energy fluence = energy per unit area
- Define:
  - "stored fluence" $\Gamma_{stor} = \frac{E_{stor}}{A}$
  - "saturation fluence" $\Gamma_{sat} = \frac{h\nu_{21}}{\sigma_{21}}$

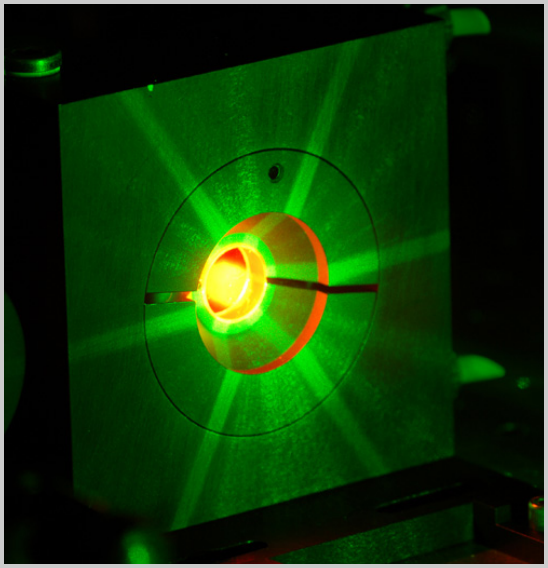
$$G_0 = \exp\left[\frac{\Gamma_{stor}}{\Gamma_{sat}}\right]$$

Example: Ti:sapphire amplifier

- Pump laser has 10mJ per pulse, calculate spot size in crystal for $G_0 = 5$

  Ti:sapphire:

  - λ_{21}=800nm, hv = 1.55eV = $2.48\text{x}10^{-19}$ J
  - $\sigma_{21} = 2.8 \text{ x } 10^{-19}$ cm^2
  - $\Gamma_{sat} = 0.85$ J/cm^2



$\Gamma_{stor} = \Gamma_{sat} \ln[G_0] = 1.37$ J/cm^2

$A = 7.3 \text{ x } 10^{-3}$ cm^2

$w_0 = 480$ um

For pulse duration of 10ns, pump intensity is

$I = 1.37 \text{ x } 10^8$ W/cm^2