Physical and Fourier Optics (PHGN570) posted 19 January 2016
Homework 1 due Friday 29 January 2016, 5pm

1. Use Mathematica functions to perform a set of Fourier transform pairs. For the first three, chose t_0=1 and plot the functions in both t and ω domains.
   a. $\Im\left\{\text{rect}\left[t/t_0\right]\right\}$ use UnitBox[]
   b. $\Im\left\{\exp\left[-t^2/t_0^{\,2}\right]\right\}$
   c. $\Im\left\{\Lambda\left[t/t_0\right]\right\}$ use UnitTriangle[] For this triangle function, also calculate the result by doing the direct integral manually.
   d. $\Im\left\{\exp\left[-t/b\right]e^{-i\omega_0 t}\Theta(t)\right\}$, where, $\Theta(t)=0$ (t<0) =1 (t≥0) = UnitStep[t]

      Here, plot the real part in the time domain, choosing parameters so that there are several oscillations within the damping time. Then plot the real and imaginary parts of the transform in frequency space.

2. Symmetry properties of Fourier transforms
   a. Show that if *f(t)* is odd and real, *F(ω)* is imaginary and odd.
   b. Show that if *f(t)* is real, *F(ω)* is in general complex, but is constrained by the symmetry property $F(-\omega)=F^*(\omega)$
   c. Show that when is *f(t)* is real, $\left|F(\omega)\right|^2$ is an even function of ω.

3. Consider a double rect pulse of the form
   $f(t)=rect\left[\left(t-t_1\right)/T\right]+rect\left[\left(t+t_1\right)/T\right]$
   where $T \ll t_1$, and both are real and positive. Calculate the Fourier transform $F(\omega)=\Im\left\{f(t)\right\}$ several ways. By making a plot, show they are all equivalent.
   a. By direct integration of the transform.
   b. Do this transform using the FourierTransform[ ] function in Mathematica. Our convention for the transforms requires you use the option FourierParameters $\rightarrow \{1,1\}$. UnitBox[ ] is the equivalent of rect[ ].
   c. Using linearity by subtracting a narrow rect from a larger rect.
   d. By convolving a rect with two delta functions at +/- t_1.
   e. By using the shift theorem.

4. Given that the Fourier transform of *f(t)* is *F(ω)*, find general expressions for the Fourier transforms of $g(t)=\int_0^t f(t')dt'$ and $h(t)=df/dt$ in terms of *F(ω)*. Hint: replace *f(t)* with its transform inside the expressions above.