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It is necessary, however, to double the above value to obtain the total number of modes, since for a given direction of propagation there are two orthogonal polarizations of the electromagnetic radiation inside the cavity. The final value for the number of modes $g$ per unit volume, for all frequencies equal to or less than $\nu$, is

$$g = \frac{8\pi}{3c^3}\nu^3 \tag{7.13}$$

We can now find, by differentiation, the number of modes per unit volume for frequencies lying between $\nu$ and $\nu + d\nu$. The result is

$$dg = \frac{8\pi}{c^3}\nu^2\, d\nu \tag{7.14}$$

A convenient way of interpreting the above result is to say that the number of modes per unit volume per unit frequency interval is

$$g_\nu = \frac{8\pi\nu^2}{c^3} \tag{7.15}$$

Although the above formula has been derived for a rectangular cavity, the result is independent of the shape of the cavity provided the cavity dimensions are large compared to the wavelength of the radiation.

## 7.4 Classical Theory of Blackbody Radiation. The Rayleigh-Jeans Formula

According to classical kinetic theory, the temperature of a gas is a measure of the mean thermal energy of the molecules that comprise the gas. The average energy associated with each degree of freedom of a molecule is $\frac{1}{2}kT$, where $k$ is Boltzmann's constant and $T$ is the absolute temperature. This well-known rule is called the *principle of equipartition of energy*. It applies, of course, only to systems in thermodynamic equilibrium.

Lord Rayleigh and Sir James Jeans suggested that the equipartition principle might also apply to the electromagnetic radiation in a cavity. If the radiation is in thermal equilibrium with the cavity walls, then one might reasonably expect an equipartition of energy among the cavity modes. Rayleigh and Jeans assumed that the mean energy per mode is $kT$. In effect, this assumption amounts to saying that in a given mode the electric field and the magnetic field each represent one degree of freedom. If there are $g_\nu$ modes per unit frequency interval per unit volume, then the spectral density of the radiation would be $g_\nu kT$. Hence, from Equation (7.15), we have

$$u_\nu = g_\nu kT = \frac{8\pi\nu^2 kT}{c^3} \tag{7.16}$$



This yields, in view of Equation (7.7), the following formula for the spectral radiance, that is, the power per unit area per unit frequency interval:

$$I_\nu = \frac{2\pi\nu^2 kT}{c^2} \tag{7.17}$$

This is the famous Rayleigh-Jeans formula. It predicts a frequency-squared dependence for the spectral distribution of blackbody radiation (Figure 7.5). For sufficiently low frequencies the formula is found to agree quite well with experimental data. However, at higher and higher frequencies the formula predicts that a blackbody will emit more and more radiation. This, of course, is in contradiction with observation. It is the so-called "ultraviolet catastrophe" of the classical radiation theory. The ultraviolet catastrophe clearly shows that there is a fundamental error in the classical approach.

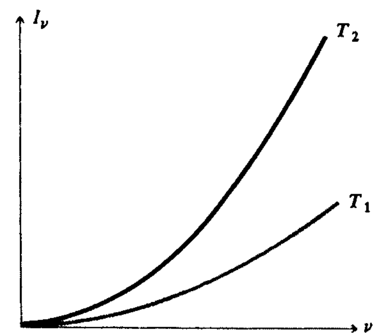


**Figure 7.5.** The Rayleigh-Jeans law. The curves of $I_\nu$ versus $\nu$ are parabolas.

## 7.5 Quantization of Cavity Radiation

The way to avert the ultraviolet catastrophe was discovered by Planck in 1901. By introducing a radical concept, namely, the *quantization* of electromagnetic radiation, Planck was able to derive an equation for blackbody emission that was in complete accord with experimental observations. This marked the beginning of quantum theory.

Planck did not accept the principle of equipartition for cavity radiation. He assumed that the energy associated with each mode was quantized; that is, the energy could exist only in integral multiples of some lowest amount or *quantum*. He postulated that the energy of the quantum was proportional to the frequency of the radiation. The name given to the quantum of electromagnetic radiation is the *photon*.