PHGN 462 Homework 1

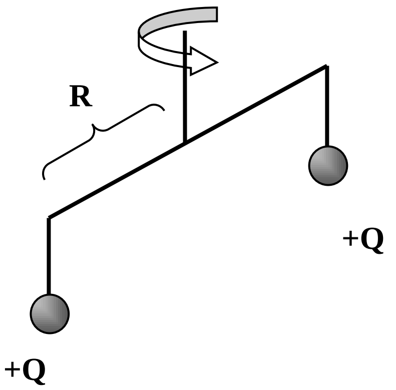
1) Take a nice, long solenoid of radius a and coil density n. It's long enough that it's a pretty good approximation to call it infinite.

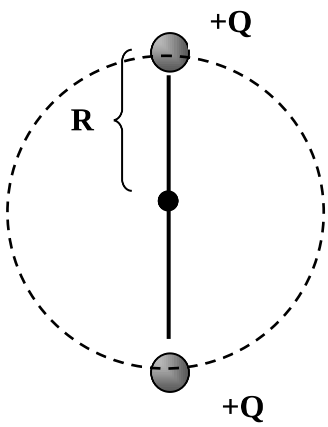
a) If there's some steady current I_0 running through it, what is the B-field inside and outside the solenoid? You don't have to derive it – just write it down.

b) Now suppose that that current is varying in time. In particular $I(t) = I_0 + kt$ for some positive constant t. Derive an expression for the E-field (if any) present in the system, both for $r < a$ and for $r > a$. Make sure you specify the direction of the field in addition to the magnitude.

c) Explicitly calculate the curl of your E-field inside and outside the solenoid and comment on what you get. Is it consistent with Faraday's law in differential form?

2) The picture shows two balls hanging from a crossbar that's free to rotate (like in a mobile, if anyone knows what those are anymore). Each ball has some positive charge Q and mass M. The crossbar is of total length 2R. There's a perspective view (left) and a top-down view (right). The arrow shows one possible rotation; it's free to rotate in either direction.





Now let's suppose the region inside the dotted circle is filled with a uniform magnetic field **B** oriented into the page. That field is turned off linearly over some time interval of length T (that is, B as a function of time is a linear function, and takes T to go from max value to zero). Describe the ensuing motion of the balls, and calculate their final angular momentum

Since we started with no motion and ended with motion, and changed only the fields, we kind of have to conclude that fields, even *static* ones, can have both linear and angular momentum if we want those quantities to be conserved (and just to be clear, we want that very much). We'll be talking about field momentum in more detail soon enough.

3) (from Pollack and Stump 10.34)

a) Show that the differential equation describing a simple RLC circuit driven by an AC voltage is:

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = V_0 \sin \omega t$$

Where Q is the charge on the capacitor and ω is the frequency of the driving voltage.

b) Derive the steady state solution *Q(t)* for this differential equation. Feel free to do it analytically or in Mathematica, but show the code if you do the latter. If you do use Mathematica, make sure the output you get is clear and interpretable, and not a six-page jumble of conditionals and such. Also, whatever you do, verify that in the limit as $R \to 0$, you get

$$Q(t) = \frac{V_0/L}{(\omega_0^2 - \omega^2)} \sin \omega t$$

Where $\omega_0 = \frac{1}{\sqrt{LC}}$, the resonant frequency for an RL circuit.

c) Now let $L = 1$, $C = 1$, and $R = 0.1$, for some arbitrary system of units. Solve our differential equation numerically, for initial values $Q(0) = 0$ and $I(0) = 0$, and driving frequency $\omega = 0.5$. You should see that after a few cycles, the solution settles down into an oscillation with the same frequency as the driving source.

d) Make plots of *Q(t)* for a few different ω's to show that for ω near ω_0, you get resonant behavior. Also plot your solution and the driving signal on the same axes and confirm that the driving signal and the response are out of phase with one another.