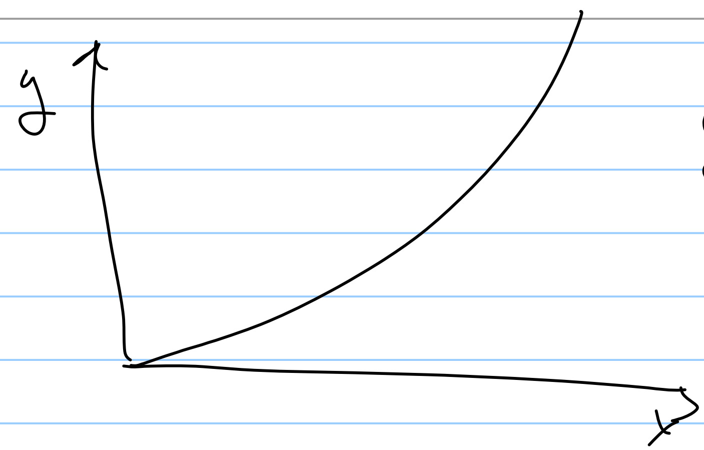
Lecture 11

Note Title 2/6/2006



$y = x^2$

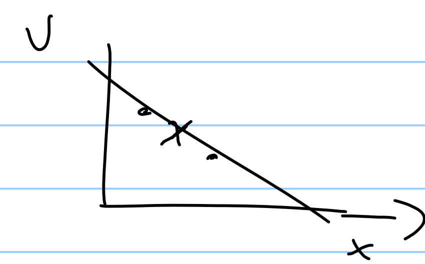
Student Answer

$d\vec{l} = dx\,\hat{x} + dy\,\hat{y}$ $\quad y = x^2;\ dy = 2x\,dx$

$L = \int |d\vec{l}| = \int_1^6 \sqrt{dx^2 + 4x^2 dx^2}$

$L = \int_1^6 \sqrt{1+4x^2}\,dx$

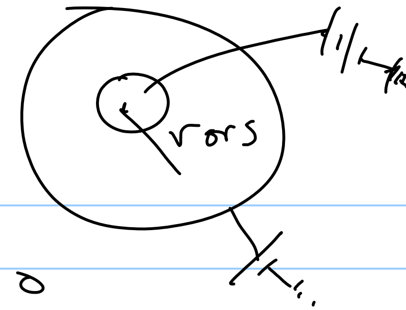
Laplace's Eqn $\quad \nabla^2 V = 0$



1-D $\quad \frac{d^2V}{dx^2} = 0 \quad V = ax + b$

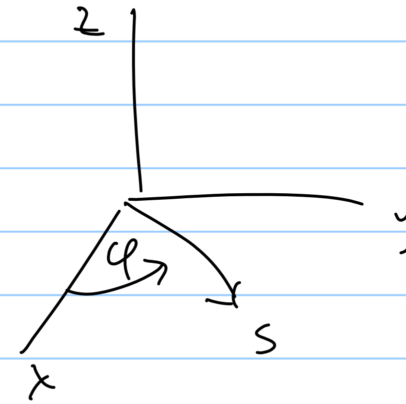
2-D

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$



$$\frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial \varphi^2} + \frac{\partial^2 t}{\partial z^2} = 0$$

$$t \longleftrightarrow V$$



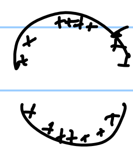
$V(s)$ $\qquad \frac{\partial V}{\partial \varphi} = 0 \qquad \frac{\partial V}{\partial z} = 0$

$$\frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial V}{\partial s}\right) = 0$$

$$V = A + B\ln(s)$$

bndry condition determine 2 constants

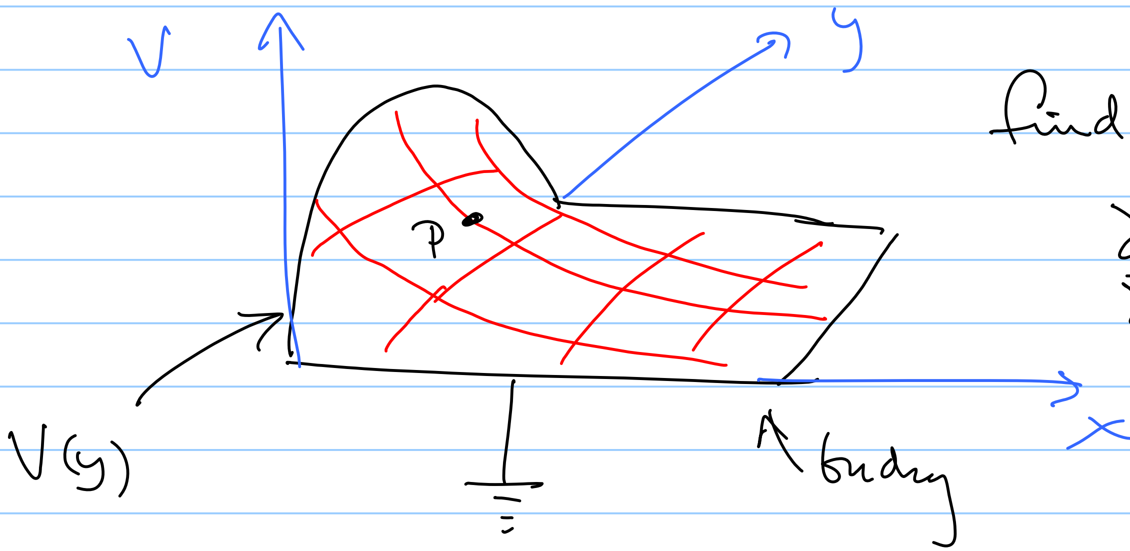
2-D



not a conductor

put some charge distribution

For a true 2-D problem you have ∞ # of bndry pts to satisfy in addition to satisfying $\nabla^2 V = 0$



find $V(x,y)$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

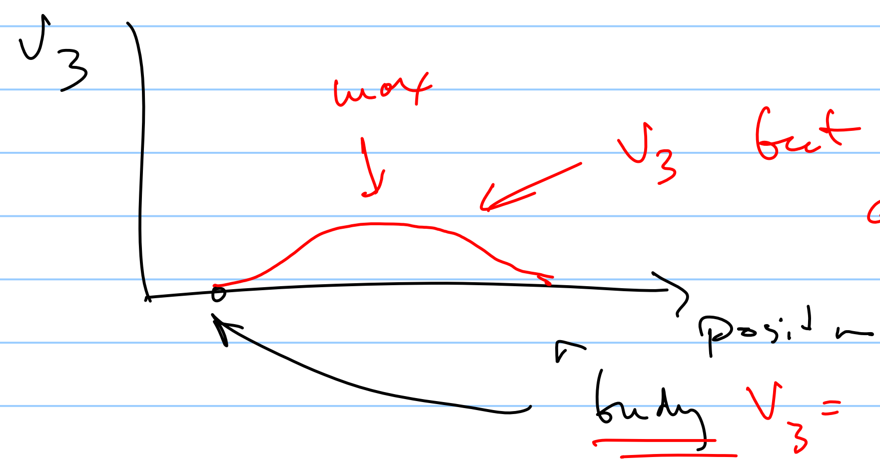
Uniqueness Theorem

Satisfy Poisson's Eqn & bndry

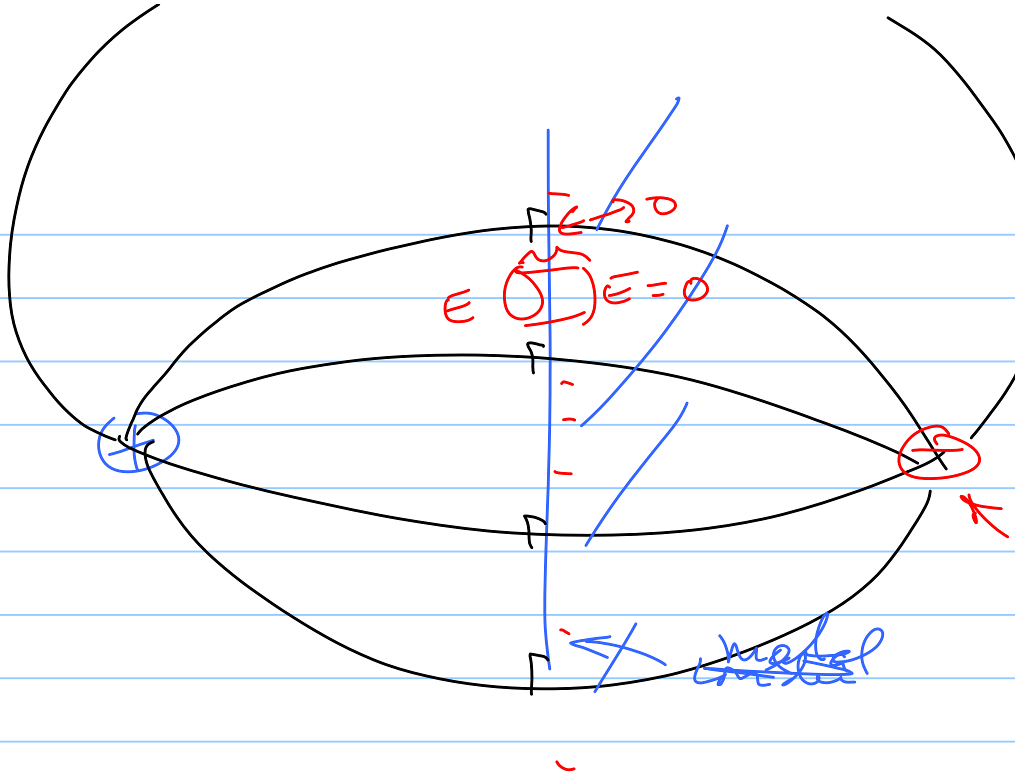
$$\nabla^2 V_1 = -\frac{\rho}{\epsilon_0} \qquad \nabla^2 V_2 = -\frac{\rho}{\epsilon_0}$$

Define $V_3 = V_1 - V_2$ also satisfies $\nabla^2 V_3 = \frac{-\rho}{\epsilon_0} - -\frac{\rho}{\epsilon_0}$

$= 0$



but V_3 can only max or min at bndry



$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}$$

$$E\cancel{A} = \frac{\sigma \cancel{A}}{\epsilon_0}$$

image charge

$$E = \frac{\sigma}{\epsilon_0}$$

calc. easily

metal surface

$$Q_{\text{induced on } \infty \text{ plane}} = \int \sigma(x,y)\, dx\, dy$$