

Scattering + dispersion theory

We have seen that oscillating currents \rightarrow radiated EM waves. Scattering arises when oscillations are driven by an incident wave.

Overview:

- free electron scattering:
 - low freq. limit = Thomson scattering (elastic)
 - high freq. \rightarrow Compton scattering (inelastic)
- bound electrons (resonant frequencies) $\rightarrow \Delta\omega$
 - \rightarrow Rayleigh scatt. (elastic) $\uparrow\downarrow$ ground
 - \rightarrow Raman scatt. (inelastic) $\uparrow\downarrow$
- collective effects
 - scattering $< \lambda \rightarrow$ coherent forward scattering
 - sum scattered + incident waves
 - \rightarrow net plane wave w/ diff't ν_{phase}
 - \rightarrow refractive index.
- high density (solid/liquid): local field effects.

classical model:

$$\text{eqn of motion for electron } \vec{F} = m\ddot{\vec{x}} = -e\vec{E}$$

w/ $\vec{E} = \vec{E}_0 e^{i(kz - \omega t)}$ = driving field,

\rightarrow SHO binding $K\ddot{x} = m\omega_0^2 x$

\rightarrow damping $\gamma\dot{x}$ radiative

$m\ddot{x}$ collisional

Thomson scattering: free electron, elastic (no Doppler shift)

$$\vec{\ddot{p}}(t) = q\vec{v}(t) = \frac{q^2 \vec{E}(t)}{m} \quad \text{note: no w-dep. in } \vec{\ddot{p}}$$

radiated power

$$\langle \frac{dP}{d\Omega} \rangle = \left\langle [\vec{\ddot{p}}]^2 \right\rangle \sin^2 \theta = \frac{1}{2} \frac{q^4 E_0^2}{m^2} \cdot \frac{\sin^2 \theta}{4\pi c^2}$$

Lamour

$$\begin{aligned} q &= e \\ m &= m_e \\ &= \left(\frac{e^2}{mc^2} \right)^2 \cdot \frac{c}{8\pi} E_0^2 \cdot \sin^2 \theta \\ &= r_e^2 I_0 \sin^2 \theta \end{aligned}$$

by classical el. radius in SI $\frac{e^2}{4\pi \epsilon_0 m_e c^2}$

diff. cross-section

$$\frac{d\sigma}{d\Omega} \equiv \frac{\langle dP/d\Omega \rangle}{I} = r_e^2 \sin^2 \theta$$

total cross-section

$$\sigma = 2\pi \int_0^\pi r_e^2 \sin^2 \theta \sin \theta d\theta$$

$$= \frac{8}{3} \cdot \pi r_e^2 \quad \text{w-indep}$$

Rayleigh scattering from bound electrons depends on w

for unpolarized light, avg. over \vec{E} directions

Can use Thomson scatt. \rightarrow diagnostic of Te in plasmas
 - Doppler broadening of incident bandwidth.

Refractive index of gases:

- transition from micro- to macroscopic description.

$$\vec{D} = \epsilon \vec{E} \text{ in a linear medium.}$$

$\epsilon = n^2$ comes from medium response.

Applied field induces a polarization in the medium:

$$\vec{P} = \sum_{\alpha} N_{\alpha} \vec{p}_{\alpha}$$

α = species index

N_{α} = number density

(book: $N f_{\alpha}$)

$$\vec{D} = \vec{E} + 4\pi \vec{P}$$

explicitly shows how medium responds
if $\vec{P} \propto \vec{E} \rightarrow$ linear, isotropic

$$\rightarrow \vec{P} = \chi_e \vec{E}$$

$$\vec{D} = \underbrace{(1 + 4\pi \chi_e)}_{\epsilon} \vec{E}$$

absorption $\rightarrow \chi_e$ is complex.

if $\vec{P} \neq \vec{E}$, \vec{D} is redirected.

\rightarrow birefringence

$\epsilon \rightarrow \epsilon$ tensor

if response is nonlinear, e.g. anharmonic osc.

$$\text{write } P = \chi^{(1)}_e E + \frac{1}{2} \chi^{(2)}_e E^2 + \frac{1}{3!} \chi^{(3)}_e E^3 + \dots$$

\rightarrow nonlinear optics.

$$\text{e.g. } P_{NL} = \frac{1}{2} \chi^{(2)}_e (E_0 \cos \omega t)^2 \text{ oscillates at } 2\omega \\ \rightarrow \text{SHG.}$$

Our procedure:

$$\text{calculate } r'(t) \rightarrow P \rightarrow \vec{P} = \chi_e \vec{E} \rightarrow \epsilon = 1 + 4\pi \chi_e$$