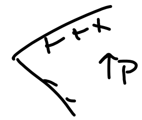
Lecture 19



Note Title 3/1/2006

Let $\vec{p} = p_y \hat{y}$ compare $\begin{cases} (\vec{p}\cdot\vec{\nabla})\vec{E} & (1) \\ \vec{\nabla}(\vec{p}\cdot\vec{E}) & (2) \end{cases}$

Product rule (4)

$$\vec{\nabla}(\underset{\uparrow \vec{p}}{\vec{A}}\cdot\underset{\uparrow \vec{E}}{\vec{B}}) = \vec{A}\times(\vec{\nabla}\times\vec{B}) + \vec{B}\times(\vec{\nabla}\times\vec{A}) + (\vec{A}\cdot\vec{\nabla})\vec{B} + (\vec{B}\cdot\vec{\nabla})\vec{A}$$

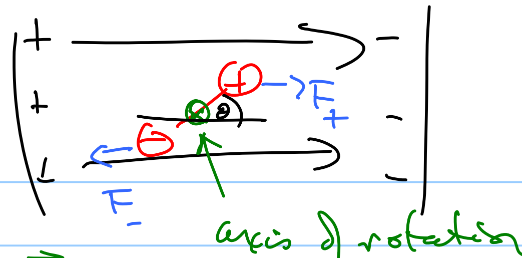
(1) $p_y \frac{\partial}{\partial y}\vec{E} = p_y \frac{\partial}{\partial y} E_x \hat{x} + p_y \frac{\partial}{\partial y} E_y \hat{y} + p_y \frac{\partial}{\partial y} E_z \hat{z}$

(2) $\vec{\nabla}(p_y E_y) = \hat{x} p_y \frac{\partial E_y}{\partial x} + \hat{y} p_y \frac{\partial E_y}{\partial y} + \hat{z} p_y \frac{\partial E_y}{\partial z}$

Now $\vec{\nabla}\times\vec{E} = 0 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} \Rightarrow \frac{\partial E_z}{\partial y} = \frac{\partial E_y}{\partial z}\ ;\ \frac{\partial E_y}{\partial z} = \frac{\partial E_z}{\partial y}$

$$\frac{\partial E_y}{\partial x} = \frac{\partial E_x}{\partial y}$$

Energy



$$\underline{\vec N} = \vec r \times \vec F_+ + (-\vec r)\times \underbrace{\vec F_-}_{= q\vec E} \rightarrow \vec p \times \vec E$$

$$W_{net} = \Delta KE$$

$$W_{non\,cons} + \underbrace{W_{cons}}_{= -\Delta PE} = \Delta KE \quad \Big\} \quad W_{ncon} = \Delta(KE + PE)$$

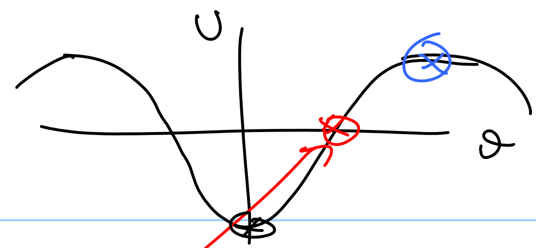
linear $\int \vec F \cdot d\vec l$

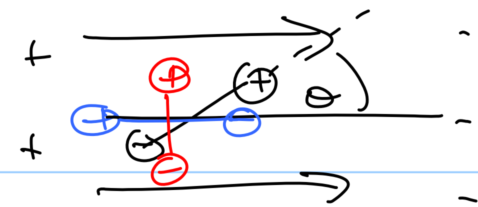
$$W_{me} = -W_{field} = -\,-(\Delta PE) = \Delta U$$

$$W_{field} = \int \vec N \cdot d\vec\theta = \int \vec p \times \vec E \cdot d\vec\theta = \int_{\theta_1}^{\theta_2} |\vec p||\vec E| \sin\theta \, d\theta$$

$$\Delta U = -W_{field} = \boxed{-\vec p \cdot \vec E} \quad -|\vec p||\vec E| \cos\theta$$

$U = 0$ when $\cos\theta = 0$

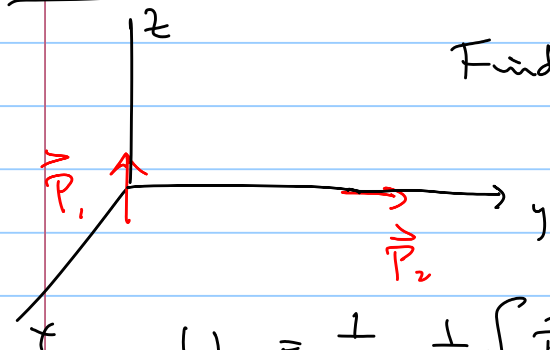




$\theta = 0 \qquad U = -pE$

$\theta = \pi/2 \qquad U = 0$

$\theta = \pi \qquad U = pE$



Find expression for $U_2 = -\vec{p}_2 \cdot \vec{E}_1$

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}\left[3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}\right]$$

$$U_2 = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}\left[\underbrace{\vec{p}_1\cdot\vec{p}_2}_{0} - 3(\vec{p}_1\cdot\hat{r})(\vec{p}_2\cdot\hat{r})\right]$$

$\vec{p}_1 = p_1\hat{z} \qquad \vec{p}_2 = p_2\hat{y}$

$$\vec{F}_2 = -\vec{\nabla} U_2$$

$$= -\left(\hat{x}\frac{\partial}{\partial x}U_2 + \hat{y}\frac{\partial}{\partial y}U_2 + \hat{z}\frac{\partial}{\partial z}U_2\right)$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{x\hat{x} + y\hat{y} + z\hat{z}}{|\vec{r}|}$$

$$U_2 = \frac{1}{4\pi\epsilon_0}\frac{1}{r^5}\left(-3P_1 z P_2 y\right) = \frac{-3P_1P_2}{4\pi\epsilon_0}\frac{yz}{(x^2+y^2+z^2)^{5/2}}$$

Cal $F = -\nabla U_2 \Big|_{\substack{x=0 \\ z=0}} =$

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other way $\vec{F}_2 = (\vec{p}_2 \cdot \vec{\nabla})\vec{E}_1 = P_2 \frac{\partial}{\partial y}\vec{E}_1$!

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}\left[\underbrace{3(\vec{p}\cdot\hat{r})}_{0}\hat{r} - \vec{p}_1\right] = \frac{-P_1}{4\pi\epsilon_0 y^3}\hat{k}$$!

$$\vec{F}_2 = P_2\frac{d}{dy}\left(\frac{-P_1\hat{k}}{4\pi\epsilon_0 y^3}\right) = \underline{\hspace{3cm}}$$