

Chapter One

INTRODUCTION

OCCURRENCE OF PLASMAS IN NATURE 1.1

It has often been said that 99% of the matter in the universe is in the plasma state; that is, in the form of an electrified gas with the atoms dissociated into positive ions and negative electrons. This estimate may not be very accurate, but it is certainly a reasonable one in view of the fact that stellar interiors and atmospheres, gaseous nebulae, and much of the interstellar hydrogen are plasmas. In our own neighborhood, as soon as one leaves the earth's atmosphere, one encounters the plasma comprising the Van Allen radiation belts and the solar wind. On the other hand, in our everyday lives encounters with plasmas are limited to a few examples: the flash of a lightning bolt, the soft glow of the Aurora Borealis, the conducting gas inside a fluorescent tube or neon sign, and the slight amount of ionization in a rocket exhaust. It would seem that we live in the 1% of the universe in which plasmas do not occur naturally.

The reason for this can be seen from the Saha equation, which tells us the amount of ionization to be expected in a gas in thermal equilibrium:

$$\frac{n_i}{n_n} \approx 2.4 \times 10^{21} \frac{T^{3/2}}{n_i} e^{-U_i/KT} \quad [1-1]$$

Here n_i and n_n are, respectively, the density (number per m^3) of ionized atoms and of neutral atoms, T is the gas temperature in $^\circ K$, K is Boltzmann's constant, and U_i is the ionization energy of the gas—that

is, the number of ergs required to remove the outermost electron from an atom. (The mks or International System of units will be used in this book.) For ordinary air at room temperature, we may take $n_n \approx 3 \times 10^{25} \text{ m}^{-3}$ (see Problem 1-1), $T \approx 300^\circ\text{K}$, and $U_i = 14.5 \text{ eV}$ (for nitrogen), where $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$. The fractional ionization $n_i/(n_n + n_i) \approx n_i/n_n$ predicted by Eq. [1-1] is ridiculously low:

$$\frac{n_i}{n_n} \approx 10^{-122}$$

As the temperature is raised, the degree of ionization remains low until U_i is only a few times KT . Then n_i/n_n rises abruptly, and the gas is in a plasma state. Further increase in temperature makes n_n less than n_i , and the plasma eventually becomes fully ionized. This is the reason plasmas exist in astronomical bodies with temperatures of millions of degrees, but not on the earth. Life could not easily coexist with a plasma—at least, plasma of the type we are talking about. The natural occurrence of plasmas at high temperatures is the reason for the designation “the fourth state of matter.”

Although we do not intend to emphasize the Saha equation, we should point out its physical meaning. Atoms in a gas have a spread of thermal energies, and an atom is ionized when, by chance, it suffers a

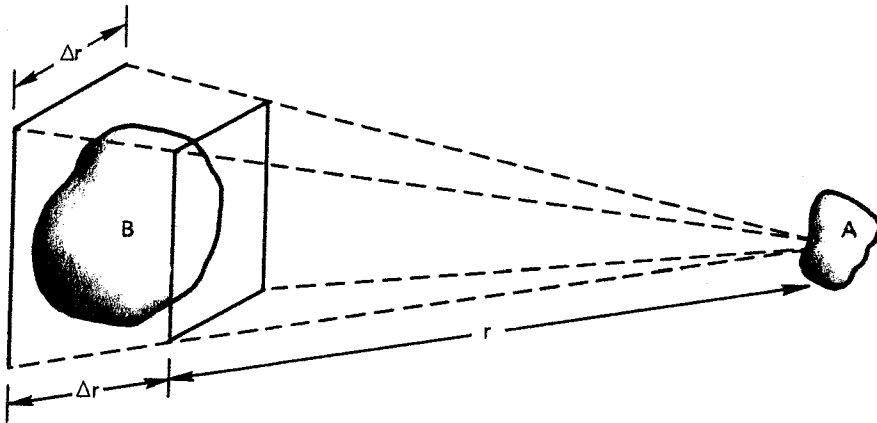


FIGURE 1-1 Illustrating the long range of electrostatic forces in a plasma.

collision of high enough energy to knock out an electron. In a cold gas, such energetic collisions occur infrequently, since an atom must be accelerated to much higher than the average energy by a series of “favorable” collisions. The exponential factor in Eq. [1-1] expresses the fact that the number of fast atoms falls exponentially with U_i/KT . Once an atom is ionized, it remains charged until it meets an electron; it then very likely recombines with the electron to become neutral again. The recombination rate clearly depends on the density of electrons, which we can take as equal to n_i . The equilibrium ion density, therefore, should decrease with n_i ; and this is the reason for the factor n_i^{-1} on the right-hand side of Eq. [1-1]. The plasma in the interstellar medium owes its existence to the low value of n_i (about 1 per cm^3), and hence the low recombination rate.

DEFINITION OF PLASMA 1.2

Any ionized gas cannot be called a plasma, of course; there is always some small degree of ionization in any gas. A useful definition is as follows:

A plasma is a quasineutral gas of charged and neutral particles which exhibits collective behavior.

We must now define “quasineutral” and “collective behavior.” The meaning of quasineutrality will be made clear in Section 1.4. What is meant by “collective behavior” is as follows.

Consider the forces acting on a molecule of, say, ordinary air. Since the molecule is neutral, there is no net electromagnetic force on it, and the force of gravity is negligible. The molecule moves undisturbed until it makes a collision with another molecule, and these collisions control the particle's motion. A macroscopic force applied to a neutral gas, such as from a loudspeaker generating sound waves, is transmitted to the individual atoms by collisions. The situation is totally different in a plasma, which has *charged* particles. As these charges move around, they can generate local concentrations of positive or negative charge, which give rise to electric fields. Motion of charges also generates currents, and hence magnetic fields. These fields affect the motion of other charged particles far away.

Let us consider the effect on each other of two slightly charged regions of plasma separated by a distance r (Fig. 1-1). The Coulomb force between A and B diminishes as $1/r^2$. However, for a given solid angle (that is, $\Delta r/r = \text{constant}$), the volume of plasma in B that can affect

A increases as r^3 . Therefore, elements of plasma exert a force on one another even at large distances. It is this long-ranged Coulomb force that gives the plasma a large repertoire of possible motions and enriches the field of study known as plasma physics. In fact, the most interesting results concern so-called "collisionless" plasmas, in which the long-range electromagnetic forces are so much larger than the forces due to ordinary local collisions that the latter can be neglected altogether. By "collective behavior" we mean motions that depend not only on local conditions but on the state of the plasma in remote regions as well.

The word "plasma" seems to be a misnomer. It comes from the Greek *πλάσμα*, *-ατος*, *τός*, which means something molded or fabricated. Because of collective behavior, a plasma does not tend to conform to external influences; rather, it often behaves as if it had a mind of its own.

1.3 CONCEPT OF TEMPERATURE

Before proceeding further, it is well to review and extend our physical notions of "temperature." A gas in thermal equilibrium has particles of all velocities, and the most probable distribution of these velocities is known as the Maxwellian distribution. For simplicity, consider a gas in which the particles can move only in one dimension. (This is not entirely frivolous; a strong magnetic field, for instance, can constrain electrons to move only along the field lines.) The one-dimensional Maxwellian distribution is given by

$$f(u) = A \exp(-\frac{1}{2}mu^2/KT) \quad [1-2]$$

where $f du$ is the number of particles per m^3 with velocity between u and $u + du$, $\frac{1}{2}mu^2$ is the kinetic energy, and K is Boltzmann's constant,

$$K = 1.38 \times 10^{-23} \text{ J/}^\circ\text{K}$$

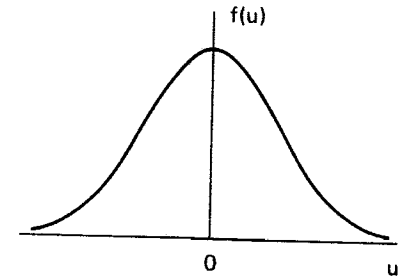
The density n , or number of particles per m^3 , is given by (see Fig. 1-2)

$$n = \int_{-\infty}^{\infty} f(u) du \quad [1-3]$$

The constant A is related to the density n by (see Problem 1-2)

$$A = n \left(\frac{m}{2\pi KT} \right)^{1/2} \quad [1-4]$$

The width of the distribution is characterized by the constant T , which we call the temperature. To see the exact meaning of T , we can



A Maxwellian velocity distribution. FIGURE 1-2

compute the average kinetic energy of particles in this distribution:

$$E_{av} = \frac{\int_{-\infty}^{\infty} \frac{1}{2}mu^2 f(u) du}{\int_{-\infty}^{\infty} f(u) du} \quad [1-5]$$

Defining

$$v_{th} = (2KT/m)^{1/2} \quad \text{and} \quad y = u/v_{th} \quad [1-6]$$

we can write Eq. [1-2] as

$$f(u) = A \exp(-u^2/v_{th}^2)$$

and Eq. [1-5] as

$$E_{av} = \frac{\frac{1}{2}mAv_{th}^3 \int_{-\infty}^{\infty} [\exp(-y^2)]y^2 dy}{Av_{th} \int_{-\infty}^{\infty} \exp(-y^2) dy}$$

The integral in the numerator is integrable by parts:

$$\begin{aligned} \int_{-\infty}^{\infty} y \cdot [\exp(-y^2)]y dy &= [-\frac{1}{2}[\exp(-y^2)]y]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} -\frac{1}{2} \exp(-y^2) dy \\ &= \frac{1}{2} \int_{-\infty}^{\infty} \exp(-y^2) dy \end{aligned}$$

Canceling the integrals, we have

$$E_{av} = \frac{\frac{1}{2}mAv_{th}^3 \frac{1}{2}}{Av_{th}} = \frac{1}{4}mv_{th}^2 = \frac{1}{2}KT \quad [1-7]$$

Thus the average kinetic energy is $\frac{1}{2}KT$.

It is easy to extend this result to three dimensions. Maxwell's distribution is then

$$f(u, v, w) = A_3 \exp[-\frac{1}{2}m(u^2 + v^2 + w^2)/KT] \quad [1-8]$$

where

$$A_3 = n \left(\frac{m}{2\pi KT} \right)^{3/2} \quad [1-9]$$

The average kinetic energy is

$$E_{av} = \frac{\iiint_{-\infty}^{\infty} A_3 \frac{1}{2}m(u^2 + v^2 + w^2) \exp[-\frac{1}{2}m(u^2 + v^2 + w^2)/KT] du dv dw}{\iiint_{-\infty}^{\infty} A_3 \exp[-\frac{1}{2}m(u^2 + v^2 + w^2)/KT] du dv dw}$$

We note that this expression is symmetric in u , v , and w , since a Maxwellian distribution is isotropic. Consequently, each of the three terms in the numerator is the same as the others. We need only to evaluate the first term and multiply by three:

$$E_{av} = \frac{3A_3 \int \frac{1}{2}mu^2 \exp(-\frac{1}{2}mu^2/KT) du \iint \exp[-\frac{1}{2}m(v^2 + w^2)/KT] dv dw}{A_3 \int \exp(-\frac{1}{2}mu^2/KT) du \iint \exp[-\frac{1}{2}m(v^2 + w^2)/KT] dv dw}$$

Using our previous result, we have

$$E_{av} = \frac{3}{2}KT \quad [1-10]$$

The general result is that E_{av} equals $\frac{1}{2}KT$ per degree of freedom.

Since T and E_{av} are so closely related, it is customary in plasma physics to give temperatures in units of energy. To avoid confusion on the number of dimensions involved, it is not E_{av} but the energy corresponding to KT that is used to denote the temperature. For $KT = 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$, we have

$$T = \frac{1.6 \times 10^{-19}}{1.38 \times 10^{-23}} = 11,600$$

Thus the conversion factor is

$$1 \text{ eV} = 11,600^\circ\text{K} \quad [1-11]$$

By a 2-eV plasma we mean that $KT = 2 \text{ eV}$, or $E_{av} = 3 \text{ eV}$ in three dimensions.

It is interesting that a plasma can have several temperatures at the same time. It often happens that the ions and the electrons have separate

Maxwellian distributions with different temperatures T_i and T_e . This can come about because the collision rate among ions or among electrons themselves is larger than the rate of collisions between an ion and an electron. Then each species can be in its own thermal equilibrium, but the plasma may not last long enough for the two temperatures to equalize. When there is a magnetic field \mathbf{B} , even a single species, say ions, can have two temperatures. This is because the forces acting on an ion along \mathbf{B} are different from those acting perpendicular to \mathbf{B} (due to the Lorentz force). The components of velocity perpendicular to \mathbf{B} and parallel to \mathbf{B} may then belong to different Maxwellian distributions with temperatures T_\perp and T_\parallel .

Before leaving our review of the notion of temperature, we should dispel the popular misconception that high temperature necessarily means a lot of heat. People are usually amazed to learn that the electron temperature inside a fluorescent light bulb is about $20,000^\circ\text{K}$. "My, it doesn't feel that hot!" Of course, the heat capacity must also be taken into account. The density of electrons inside a fluorescent tube is much less than that of a gas at atmospheric pressure, and the total amount of heat transferred to the wall by electrons striking it at their thermal velocities is not that great. Everyone has had the experience of a cigarette ash dropped innocuously on his hand. Although the temperature is high enough to cause a burn, the total amount of heat involved is not. Many laboratory plasmas have temperatures of the order of $1,000,000^\circ\text{K}$ (100 eV), but at densities of 10^{18} – 10^{19} per m^3 , the heating of the walls is not a serious consideration.

1-1. Compute the density (in units of m^{-3}) of an ideal gas under the following conditions: **PROBLEMS**

(a) At 0°C and 760 Torr pressure (1 Torr = 1 mm Hg). This is called the Loschmidt number.

(b) In a vacuum of 10^{-3} Torr at room temperature (20°C). This number is a useful one for the experimentalist to know by heart (10^{-3} Torr = 1 micron).

1-2. Derive the constant A for a normalized one-dimensional Maxwellian distribution

$$\hat{f}(u) = A \exp(-mu^2/2KT)$$

such that

$$\int_{-\infty}^{\infty} \hat{f}(u) du = 1$$

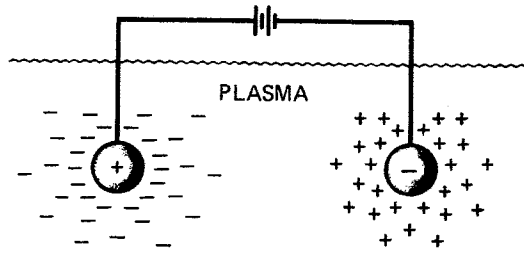
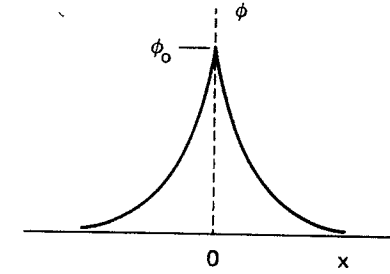


FIGURE 1-3 Debye shielding.

1.4 DEBYE SHIELDING

A fundamental characteristic of the behavior of a plasma is its ability to shield out electric potentials that are applied to it. Suppose we tried to put an electric field inside a plasma by inserting two charged balls connected to a battery (Fig. 1-3). The balls would attract particles of the opposite charge, and almost immediately a cloud of ions would surround the negative ball and a cloud of electrons would surround the positive ball. (We assume that a layer of dielectric keeps the plasma from actually recombining on the surface, or that the battery is large enough to maintain the potential in spite of this.) If the plasma were cold and there were no thermal motions, there would be just as many charges in the cloud as in the ball; the shielding would be perfect, and no electric field would be present in the body of the plasma outside of the clouds. On the other hand, if the temperature is finite, those particles that are at the edge of the cloud, where the electric field is weak, have enough thermal energy to escape from the electrostatic potential well. The "edge" of the cloud then occurs at the radius where the potential energy is approximately equal to the thermal energy KT of the particles, and the shielding is not complete. Potentials of the order of KT/e can leak into the plasma and cause finite electric fields to exist there.

Let us compute the approximate thickness of such a charge cloud. Imagine that the potential ϕ on the plane $x = 0$ is held at a value ϕ_0 by a perfectly transparent grid (Fig. 1-4). We wish to compute $\phi(x)$. For simplicity, we assume that the ion-electron mass ratio M/m is infinite, so that the ions do not move but form a uniform background of positive charge. To be more precise, we can say that M/m is large enough that



Potential distribution near a grid in a plasma. FIGURE 1-4

the inertia of the ions prevents them from moving significantly on the time scale of the experiment. Poisson's equation in one dimension is

$$\epsilon_0 \nabla^2 \phi = \epsilon_0 \frac{d^2 \phi}{dx^2} = -e(n_i - n_e) \quad (Z = 1) \quad [1-12]$$

If the density far away is n_∞ , we have

$$n_i = n_\infty$$

In the presence of a potential energy $q\phi$, the electron distribution function is

$$f(u) = A \exp[-(\frac{1}{2}mu^2 + q\phi)/KT_e]$$

It would not be worthwhile to prove this here. What this equation says is intuitively obvious: There are fewer particles at places where the potential energy is large, since not all particles have enough energy to get there. Integrating $f(u)$ over u , setting $q = -e$, and noting that $n_e(\phi \rightarrow 0) = n_\infty$, we find

$$n_e = n_\infty \exp(e\phi/KT_e)$$

This equation will be derived with more physical insight in Section 3.5. Substituting for n_i and n_e in Eq. [1-12], we have

$$\epsilon_0 \frac{d^2 \phi}{dx^2} = en_\infty \left\{ \left[\exp\left(\frac{e\phi}{KT_e}\right) \right] - 1 \right\}$$

In the region where $|e\phi/KT_e| \ll 1$, we can expand the exponential in a Taylor series:

$$\epsilon_0 \frac{d^2 \phi}{dx^2} = en_\infty \left[\frac{e\phi}{KT_e} + \frac{1}{2} \left(\frac{e\phi}{KT_e} \right)^2 + \dots \right] \quad [1-13]$$

No simplification is possible for the region near the grid, where $|e\phi/KT_e|$ may be large. Fortunately, this region does not contribute much to the thickness of the cloud (called a sheath), because the potential falls very rapidly there. Keeping only the linear terms in Eq. [1-13], we have

$$\epsilon_0 \frac{d^2\phi}{dx^2} = \frac{n_\infty e^2}{KT_e} \phi \quad [1-14]$$

Defining

$$\lambda_D \equiv \left(\frac{\epsilon_0 KT_e}{n e^2} \right)^{1/2} \quad [1-15]$$

where n stands for n_∞ , we can write the solution of Eq. [1-14] as

$$\phi = \phi_0 \exp(-|x|/\lambda_D) \quad [1-16]$$

The quantity λ_D , called the Debye length, is a measure of the shielding distance or thickness of the sheath.

Note that as the density is increased, λ_D decreases, as one would expect, since each layer of plasma contains more electrons. Furthermore, λ_D increases with increasing KT_e . Without thermal agitation, the charge cloud would collapse to an infinitely thin layer. Finally, it is the *electron* temperature which is used in the definition of λ_D because the electrons, being more mobile than the ions, generally do the shielding by moving so as to create a surplus or deficit of negative charge. Only in special situations is this not true (see Problem 1-5).

The following are useful forms of Eq. [1-15]:

$$\begin{aligned} \lambda_D &= 69(T/n)^{1/2} \text{ m,} & T \text{ in } ^\circ\text{K} \\ \lambda_D &= 7430(KT/n)^{1/2} \text{ m,} & KT \text{ in eV} \end{aligned} \quad [1-17]$$

We are now in a position to define "quasineutrality." If the dimensions L of a system are much larger than λ_D , then whenever local concentrations of charge arise or external potentials are introduced into the system, these are shielded out in a distance short compared with L , leaving the bulk of the plasma free of large electric potentials or fields. Outside of the sheath on the wall or on an obstacle, $\nabla^2\phi$ is very small, and n_i is equal to n_e , typically, to better than one part in 10^6 . It takes only a small charge imbalance to give rise to potentials of the order of KT/e . The plasma is "quasineutral"; that is, neutral enough so that one can take $n_i = n_e = n$, where n is a common density called the *plasma*

density, but not so neutral that all the interesting electromagnetic forces vanish.

A criterion for an ionized gas to be a plasma is that it be dense enough that λ_D is much smaller than L .

The phenomenon of Debye shielding also occurs—in modified form—in single-species systems, such as the electron streams in klystrons and magnetrons or the proton beam in a cyclotron. In such cases, any local bunching of particles causes a large unshielded electric field unless the density is extremely low (which it often is). An externally imposed potential—from a wire probe, for instance—would be shielded out by an adjustment of the density near the electrode. Single-species systems, or unneutralized plasmas, are not strictly plasmas; but the mathematical tools of plasma physics can be used to study such systems.

THE PLASMA PARAMETER 1.5

The picture of Debye shielding that we have given above is valid only if there are enough particles in the charge cloud. Clearly, if there are only one or two particles in the sheath region, Debye shielding would not be a statistically valid concept. Using Eq. [1-17], we can compute the number N_D of particles in a "Debye sphere":

$$N_D = n \frac{4}{3} \pi \lambda_D^3 = 1.38 \times 10^6 T^{3/2} / n^{1/2} \quad (T \text{ in } ^\circ\text{K}) \quad [1-18]$$

In addition to $\lambda_D \ll L$, "collective behavior" requires

$$N_D \gg 1 \quad [1-19]$$

CRITERIA FOR PLASMAS 1.6

We have given two conditions that an ionized gas must satisfy to be called a plasma. A third condition has to do with collisions. The weakly ionized gas in a jet exhaust, for example, does not qualify as a plasma because the charged particles collide so frequently with neutral atoms that their motion is controlled by ordinary hydrodynamic forces rather than by electromagnetic forces. If ω is the frequency of typical plasma oscillations and τ is the mean time between collisions with neutral atoms, we require $\omega\tau > 1$ for the gas to behave like a plasma rather than a neutral gas.

The three conditions a plasma must satisfy are therefore:

1. $\lambda_D \ll L$.
2. $N_D \gg 1$.
3. $\omega\tau > 1$.

PROBLEMS

1-3. On a log-log plot of n_e vs. KT_e , with n_e from 10^6 to 10^{25} m^{-3} , and KT_e from 0.01 to 10^5 eV , draw lines of constant λ_D and N_D . On this graph, place the following points (n in m^{-3} , KT in eV):

1. Typical fusion reactor: $n = 10^{21}$, $KT = 10,000$.
2. Typical fusion experiments: $n = 10^{19}$, $KT = 100$ (torus); $n = 10^{23}$, $KT = 1000$ (pinch).
3. Typical ionosphere: $n = 10^{11}$, $KT = 0.05$.
4. Typical glow discharge: $n = 10^{15}$, $KT = 2$.
5. Typical flame: $n = 10^{14}$, $KT = 0.1$.
6. Typical Cs plasma: $n = 10^{17}$, $KT = 0.2$.
7. Interplanetary space: $n = 10^6$, $KT = 0.01$.

Convince yourself that these are plasmas.

1-4. Compute the pressure, in atmospheres and in tons/ft², exerted by a thermonuclear plasma on its container. Assume $KT_e = KT_i = 20 \text{ keV}$, $n = 10^{21} \text{ m}^{-3}$, and $p = nKT$, where $T = T_i + T_e$.

1-5. In a strictly steady state situation, both the ions and the electrons will follow the Boltzmann relation

$$n_j = n_0 \exp(-q_j \phi / KT_j)$$

For the case of an infinite, transparent grid charged to a potential ϕ , show that the shielding distance is then given approximately by

$$\lambda_D^{-2} = \frac{ne^2}{\epsilon_0} \left(\frac{1}{KT_e} + \frac{1}{KT_i} \right)$$

Show that λ_D is determined by the temperature of the colder species.

1-6. An alternative derivation of λ_D will give further insight to its meaning. Consider two infinite, parallel plates at $x = \pm d$, set at potential $\phi = 0$. The space between them is uniformly filled by a gas of density n of particles of charge q .

(a) Using Poisson's equation, show that the potential distribution between the plates is

$$\phi = \frac{nq}{2\epsilon_0} (d^2 - x^2)$$

(b) Show that for $d > \lambda_D$, the energy needed to transport a particle from a plate to the midplane is greater than the average kinetic energy of the particles.

1-7. Compute λ_D and N_D for the following cases:

- (a) A glow discharge, with $n = 10^{16} \text{ m}^{-3}$, $KT_e = 2 \text{ eV}$.
- (b) The earth's ionosphere, with $n = 10^{12} \text{ m}^{-3}$, $KT_e = 0.1 \text{ eV}$.
- (c) A θ -pinch, with $n = 10^{23} \text{ m}^{-3}$, $KT_e = 800 \text{ eV}$.

APPLICATIONS OF PLASMA PHYSICS 1.7

Plasmas can be characterized by the two parameters n and KT_e . Plasma applications cover an extremely wide range of n and KT_e : n varies over 28 orders of magnitude from 10^6 to 10^{24} m^{-3} , and KT can vary over seven orders from 0.1 to 10^6 eV . Some of these applications are discussed very briefly below. The tremendous range of density can be appreciated when one realizes that air and water differ in density by only 10^3 , while water and white dwarf stars are separated by only a factor of 10^5 . Even neutron stars are only 10^{15} times denser than water. Yet gaseous plasmas in the entire density range of 10^{28} can be described by the same set of equations, since only the classical (non-quantum mechanical) laws of physics are needed.

Gas Discharges (Gaseous Electronics) 1.7.1

The earliest work with plasmas was that of Langmuir, Tonks, and their collaborators in the 1920's. This research was inspired by the need to develop vacuum tubes that could carry large currents, and therefore had to be filled with ionized gases. The research was done with weakly ionized glow discharges and positive columns typically with $KT_e \approx 2 \text{ eV}$ and $10^{14} < n < 10^{18} \text{ m}^{-3}$. It was here that the shielding phenomenon was discovered; the sheath surrounding an electrode could be seen visually as a dark layer. Gas discharges are encountered nowadays in mercury rectifiers, hydrogen thyatrons, ignitrons, spark gaps, welding arcs, neon and fluorescent lights, and lightning discharges.

Controlled Thermonuclear Fusion 1.7.2

Modern plasma physics had its beginnings around 1952, when it was proposed that the hydrogen bomb fusion reaction be controlled to make a reactor. The principal reactions, which involve deuterium (D) and