Phys 361 Homework 3

1) (Based on 4.1 in Pollack and Stump)

Suppose we have a chunk of conductor with a hollow cavity inside it. Neither the conductor nor the cavity has any particular symmetry. Stick some positive point charge $+q$ in the cavity (there are no other charges).

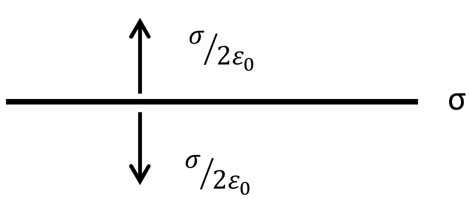
a) Prove (using lots of words) that the net charge on the wall of the cavity must be $-q$.

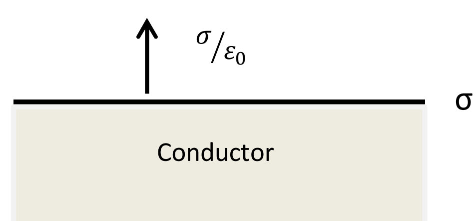
b) If the conducting object overall has a net charge of Q_0, what must be the net charge on the outer surface of the object?

c) Think about the electric field lines inside the cavity. Explain to me why it must be the case that those lines can't begin or end anywhere but the positive point charge or the wall, respectively.

Note: If you go looking for this problem in Pollack and Stump, they provide a hint involving the line integral of E. I never really figured out where they were going with that, and didn't use it personally.

2) Here's something that bothered me the first time I thought of it. Consider the case of an infinite 2-D sheet of charge with uniform charge density σ. We know that above the sheet we'll have an electric field of magnitude $\sigma/_{2\varepsilon_0}$. Now consider another system. It's an infinite block of metal with a flat surface at $z = 0$, as shown. At that surface there exists an identical charge density σ. But from what we've learned, the electric field above *that* surface must be of magnitude $\sigma/_{\varepsilon_0}$ (think about our boundary condition for $E_\perp$ to see where that comes from).





So to recap: In each physical situation we have a sheet of charge of the same magnitude. In each physical situation we have zero charge everywhere else (no free charge in the conductor). Therefore, if we did direct Coulomb's law integration to find the field in either topside region, we'd <u>get the same thing in both cases</u> ($\sigma/_{2\varepsilon_0}$; we did this in homework 1).

But somehow that's not what happens. And Coulomb's law is kind of supposed to work always. So how can this be? Don't just say "Because Gauss's Law and/or the boundary conditions demand it;" we already used Gauss's Law to find these fields. And besides, Gauss's Law and Coulomb's Law are equivalent, so having them generate different answers for the field is extra-bad. I want an explanation of how this apparent paradox can come to pass and hopefully be resolved.

3) Find the capacitance per unit length of two parallel wires, each of radius R, separated by some center-to-center distance D. Note that this is rather similar to example 6 in section 4.4 of Pollack and Stump, but there they do some pretty serious business to get the potential first. You don't need to do that. You can find the capacitance of this system using the standard Phys 200 approach and get an answer that doesn't have an arccosh in it. Note also that the interwebs provide several different, but similar, answers to this problem. Make sure your answer matches your process.