1. Assume $A = \left[-\frac{1}{2}, \infty\right)$, $B = \left[-\frac{1}{4}, \infty\right)$. Consider $f : A \to B$ defined by $f(x) = x^2 + x, x \in A$.

1. Show that f does indeed map A into B, that is, show that if $x \in A$, then $f(x) \in B$.
2. Show that f is an injection.
3. Show that f maps A onto B, that is, show that for $y \in B$, there is an $x \in A$ such that $f(x) = y$. (For your choice of x, verify directly that $f(x) = y$)

2. Definition: If $S \subset R$ and $S \neq \varnothing$, then we say that S has a maximum if and only if there is a $y \in S$ such that $x \leq y$ for every $x \in S$.
Assume $n \in \mathbb{N}$, $S_n \subset \mathbb{R}$ and S_n has n elements, that is, $S_n = \{a_1, a_2, \ldots, a_n\}$ where $a_i \in R$. Use mathematical induction on n to show that S_n has a maximum according to the definition given above.

3. Let $A = \mathbb{N} \times \mathbb{N}$, and define a relation R on A by

$$(a, b) \sim (c, d) \Leftrightarrow a^b = c^d$$

1. Show that R is an equivalence relation on A.
2. Find the equivalence class of $E_{(9,2)}$.
3. Find an equivalence class with exactly 2 elements.
4. Find an equivalence class with exactly 4 elements.

4. Suppose R is an equivalence relation on A, S is an equivalence relation on B, and $A \cap B = \varnothing$.

1. Prove that $R \cup S$ is an equivalence relation on $A \cup B$.
2. Prove that for all $x \in A$, $[x]_{R\cup S} = [x]_R$, and for all $y \in B$, $[y]_{R\cup S} = [y]_S$.
3. Prove that $(A \cup B)/(R \cup S) = (A/R) \cup (B/S)$.
   Note: (A/R) is the set of all equivalence classes of A given the equivalence relation R.