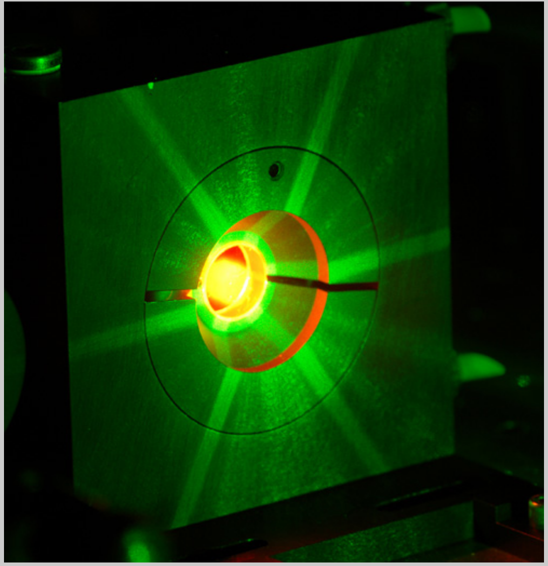
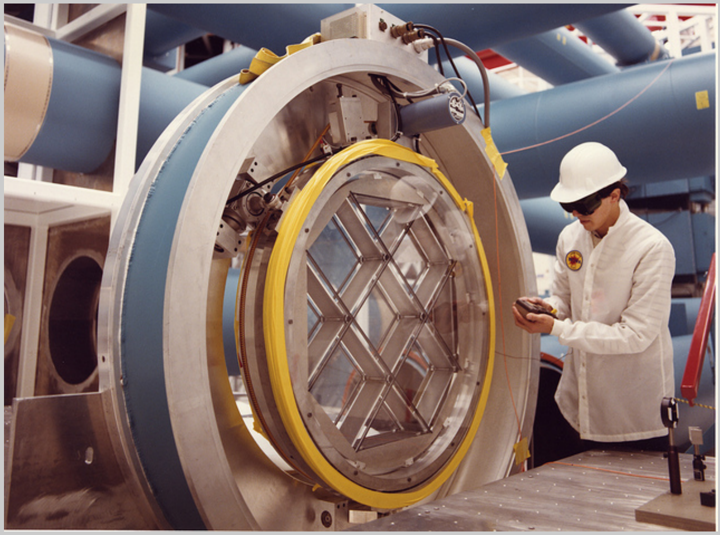
Amplifiers: pumping and small-signal gain

- Absorption $I[z]=I_0\exp[-N_0\sigma_{12}z]=I_0\exp[-\alpha z]$
- Gain $I[z]=I_0\exp[N_{inv}\sigma_{21}z]=I_0\exp[gz]$
  - What is the inversion density?
  - How to express it in terms of the pump distribution
  - How does gain depend on λ or ω ?
  - What happens when the inversion density is depleted?





Simple gain calculation

- Assume uniform pump distribution

$G_0 = \exp\left[N_{inv}\sigma_{21} L\right]$ Small-signal gain

- Available energy for extraction:

$$E_{stor} = N_{inv} A L h\nu_{21} \rightarrow N_{inv} = \frac{E_{stor}}{A L h\nu_{21}}$$

A = area of beam

$$G_0 = \exp\left[\frac{E_{stor}}{A}\frac{\sigma_{21}}{h\nu_{21}}\right]$$

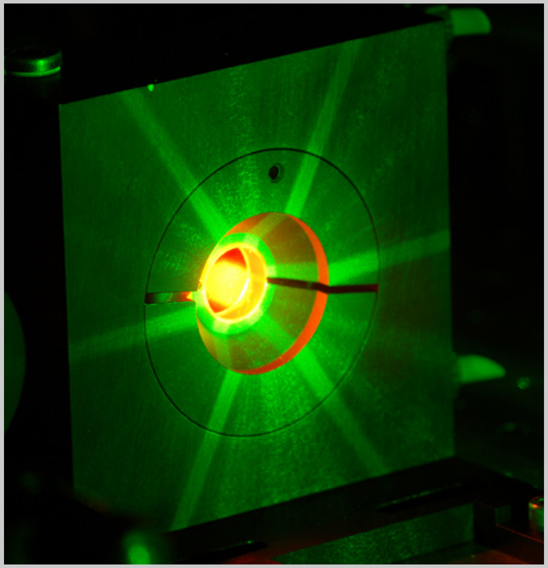
- Energy fluence = energy per unit area
- Define:
  - "stored fluence" $\Gamma_{stor} = \frac{E_{stor}}{A}$
  - "saturation fluence" $\Gamma_{sat} = \frac{h\nu_{21}}{\sigma_{21}}$

$$G_0 = \exp\left[\frac{\Gamma_{stor}}{\Gamma_{sat}}\right]$$

Example: Ti:sapphire amplifier

- Pump laser has 10mJ per pulse, calculate spot size in crystal for $G_0 = 5$

  Ti:sapphire:
  - λ_{21}=800nm, hv = 1.55eV = 2.48×10^{-19} J
  - $\sigma_{21} = 2.8 \times 10^{-19}\ \text{cm}^2$
  - $\Gamma_{sat} = 0.85\ \text{J/cm}^2$



$\Gamma_{stor} = \Gamma_{sat} \ln[G_0] = 1.37\ \text{J/cm}^2$

$A = 7.3 \times 10^{-3}\ \text{cm}^2$

$w_0 = 480$ um

For pulse duration of 10ns, pump intensity is

$I = 1.37 \times 10^{8}\ \text{W/cm}^2$

Optical pumping geometries

Technique depends on properties of light source

- Diffuse:
  - flashlamp, arc lamp (CW), LED sun
  - Pump chambers, non-imaging concentrators
- Laser beams with poor divergence:
  - laser diode elements and arrays, multi-mode fiber-coupled LDs
  - Longitudinal and side pumping
- High-quality laser beams
  - Longitudinal pumping

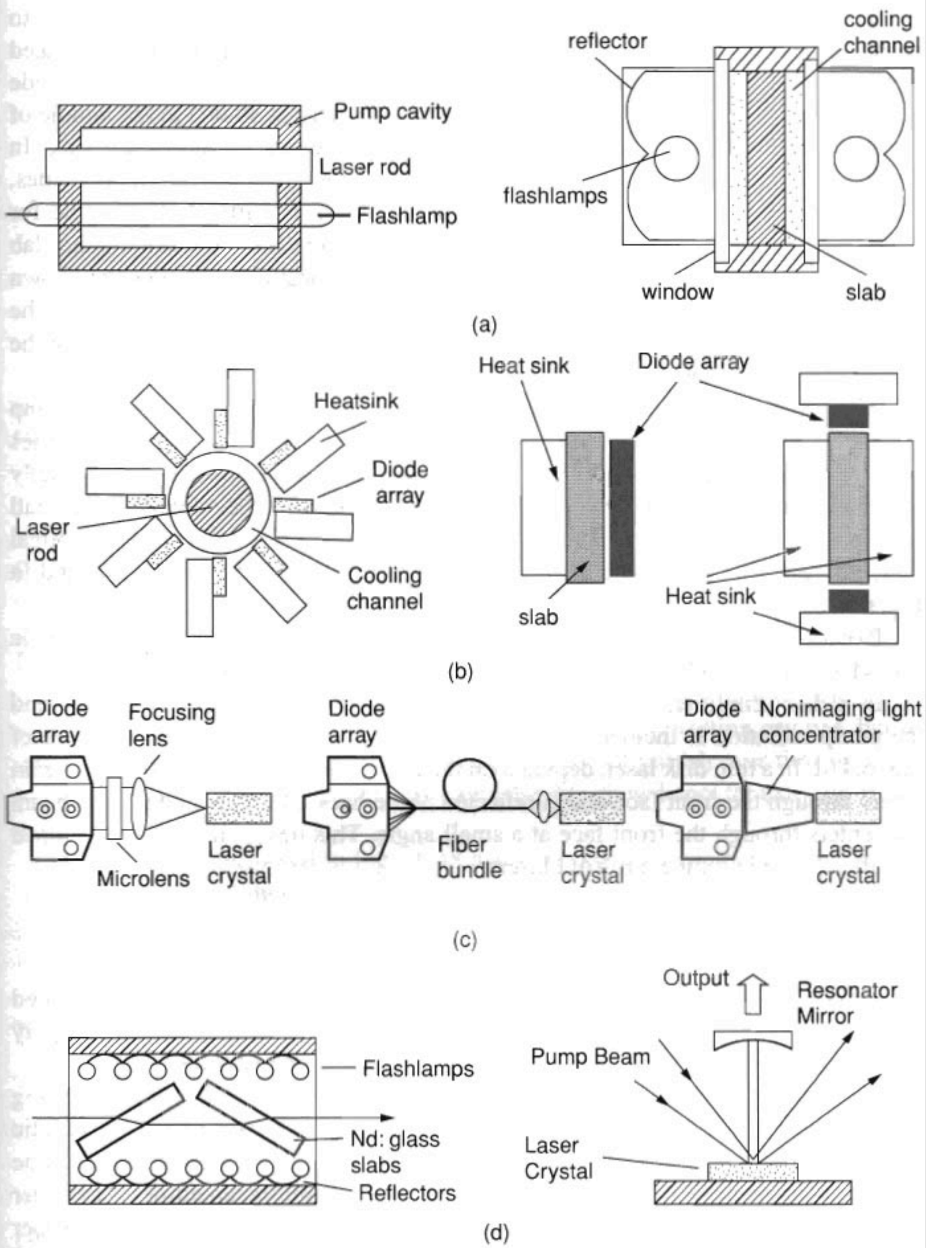
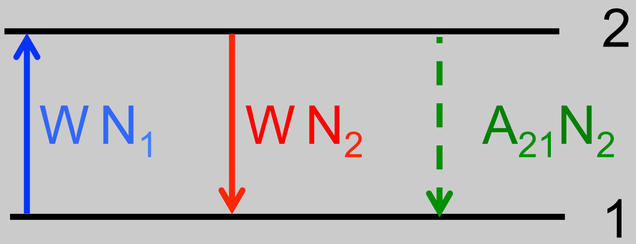


Fig. 6.47. Major pump configurations a) side pumping with flashlamps, b) side-and edge pumping with laser diodes, c) end pumped lasers, d) face pumping with flashlamps or laser diodes.

Population dynamics of absorption

- Closed 2 level system, assume $g_1=g_2$

$$\frac{dN_1}{dt} = -\frac{dN_2}{dt}$$



$$\frac{dN_2}{dt} = W\,N_1 - W\,N_2 - A_{21}N_2$$

- Since system is closed, reduce to one equation for population difference: $\Delta N = N_1 - N_2$

$$N_t = N_1 + N_2$$

$$N_t = \Delta N + 2N_2$$

$$\rightarrow N_2 = \tfrac{1}{2}\left(N_t - \Delta N\right)$$

$$\frac{dN_1}{dt} - \frac{dN_2}{dt} = \frac{d}{dt}\Delta N = -2\frac{dN_2}{dt}$$

$$\frac{d}{dt}\Delta N = -2\left(W\,\Delta N - A_{21}N_2\right)$$

$$\frac{d}{dt}\Delta N = -2W\,\Delta N + A_{21}\left(N_t - \Delta N\right) = -\Delta N\left(A_{21} + 2W\right) + A_{21}N_t$$

  - Steady state: $$\Delta N = \frac{N_t}{1 + 2W\tau_{21}} \qquad A_{21} = 1/\tau_{21}$$

Saturation of absorption

$$\boxed{\Delta N = \frac{N_t}{1 + 2W\tau_{21}}}$$

$$\Delta N = N_1 - N_2$$

- The key parameter in this situation is W τ_{21}

$$W_{21} = \rho_\nu B_{21}$$

  - Low intensity, 2W τ_{21} << 1, ΔN ≈ N_t
  - High intensity, 2W τ_{21} >> 1, ΔN ≈ 0. Here $N_1 \approx N_2$
- Energy balance:

Input power
Absorbed by atoms

Radiated power (into 4π)

Stimulated emission
(back into beam)

- Radiated power per unit volume:

$$\frac{dP}{dV} = h\nu_{21} W \Delta N(W) = h\nu_{21} \frac{N_t W}{1 + 2W\tau_{21}} \rightarrow h\nu_{21} \frac{N_t}{2\tau_{21}}$$

For W τ_{21} >> 1

Power radiated in high intensity limit: half of atoms are radiating

Saturation intensity

- Absorbed power per atom: $\sigma_{12} I$
- Absorption rate: $W = \dfrac{\sigma_{12} I}{h\nu_{21}}$
- In steady state: $\dfrac{\Delta N}{N_t} = \dfrac{1}{1+2W\tau_{21}} = \dfrac{1}{1+2\dfrac{\sigma_{12}I}{h\nu_{21}}\tau_{21}} \equiv \dfrac{1}{1+\dfrac{I}{I_{sat}}}$
- Saturation intensity for absorption: $I_{sat} = \dfrac{h\nu_{21}}{2\sigma_{12}\tau_{21}}$
  - 2: transition affects both levels at once
  - At I = I_{sat}, stimulated and spontaneous emission rates are equal.
- Intensity-dependent absorption coefficient:

$$\boxed{\alpha(I) = \frac{\alpha_0}{1+I/I_{sat}}}$$

At high intensity, material absorbs *less*.

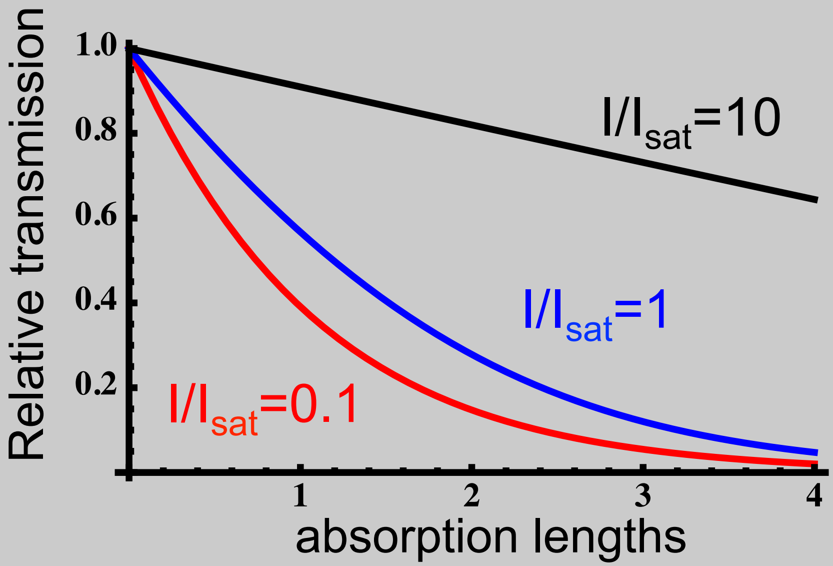
Saturable absorbers are used for pulsed lasers: Q-switching and mode-locking

Saturated CW propagation through absorbing medium

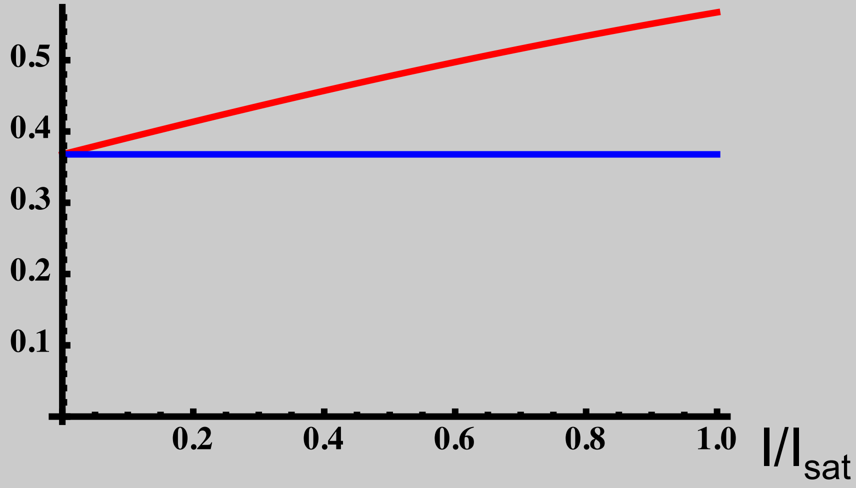
- For a given thickness for an absorbing medium, the transmission will increase with intensity

$$\boxed{\alpha(I)=\frac{\alpha_0}{1+I/I_{sat}}} \qquad \frac{dI}{dz}=-\alpha(I)I=-\frac{\alpha_0}{1+I/I_{sat}}I$$

$$\int_{I_0}^{I}\left(\frac{1}{I}+\frac{1}{I_{sat}}\right)dI=-\int_0^L\alpha_0\,dz\rightarrow\ln\left[\frac{I(z)}{I(0)}\right]+\frac{I(z)-I(0)}{I_{sat}}=-\alpha_0 z$$



Transmission over 1 absorption length



Pulsed input: saturation fluence

$$\Delta N = N_1 - N_2$$

Γ_{sat} = saturation fluence

- Rewrite equation using intensity:

$$\frac{d}{dt}\Delta N = -\Delta N\left(A_{21} + \frac{2\sigma}{h\nu_{21}} I(t)\right) + A_{21}N_t \equiv -\Delta N\left(A_{21} + \frac{I(t)}{\Gamma_{sat}}\right) + A_{21}N_t$$

- Scaling of equation
  - Two timescales: τ_p and τ_{21}, but pay attention to weighting

$$\frac{d}{dt}\Delta N = -\Delta N\left(\frac{1}{\tau_{21}} + \frac{\Gamma_{in}}{\Gamma_{sat}}\frac{1}{\tau_p}\right) + \frac{1}{\tau_{21}}N_t = \frac{2N_2}{\tau_{21}} - \frac{\Gamma_{in}}{\Gamma_{sat}}\frac{1}{\tau_p}\Delta N$$

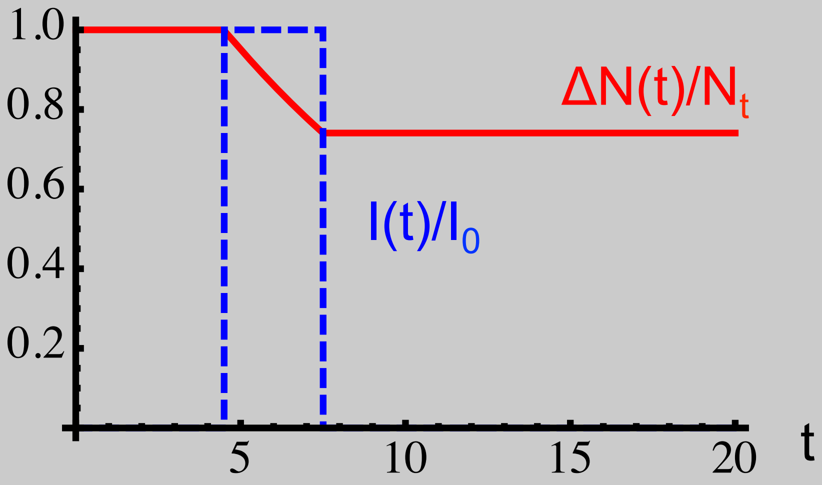
- For short pulse input: ignore stimulated emission and fluorescence

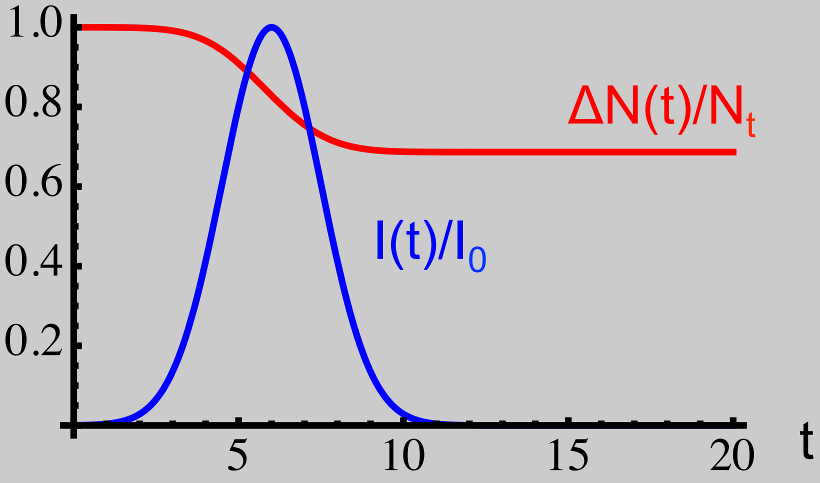
$$\frac{\Gamma_{in}}{\Gamma_{sat}}\frac{1}{\tau_p}|\Delta N| \gg \frac{2N_2}{\tau_{21}} \quad \rightarrow \frac{d}{dt}\Delta N \approx -\frac{\Gamma_{in}}{\Gamma_{sat}}\frac{1}{\tau_p}\Delta N \quad \rightarrow \ln\left[\frac{\Delta N(t)}{\Delta N(0)}\right] \approx -\frac{1}{\Gamma_{sat}}\int_0^t I(t')\,dt'$$

$$\rightarrow \Delta N(t) = N_1(t) - N_2(t) \approx N_t \exp\left[-\frac{1}{\Gamma_{sat}}\int_0^t I(t')\,dt'\right] = N_t \exp\left[-\frac{\Gamma_{in}}{\Gamma_{sat}}\right]$$

Short pulse limit

- For short pulse input: $\tau_p << \tau_{21}$, so ignore fluorescence
  - Medium just integrates energy of pulse.
  - Example: Ti:sapphire: τ_{21}=3.2μs, τ_p=10ns or 200ns for Q-switched Nd:YAG lasers pumped with flashlamps or CW arc lamps
- Square input pulse Gaussian input pulse
  - $\tau = 3$, $I_0/I_{sat} = 0.1$, (no fluorescence)





- Shape of *transmitted* pulse is affected

Long pulse limit

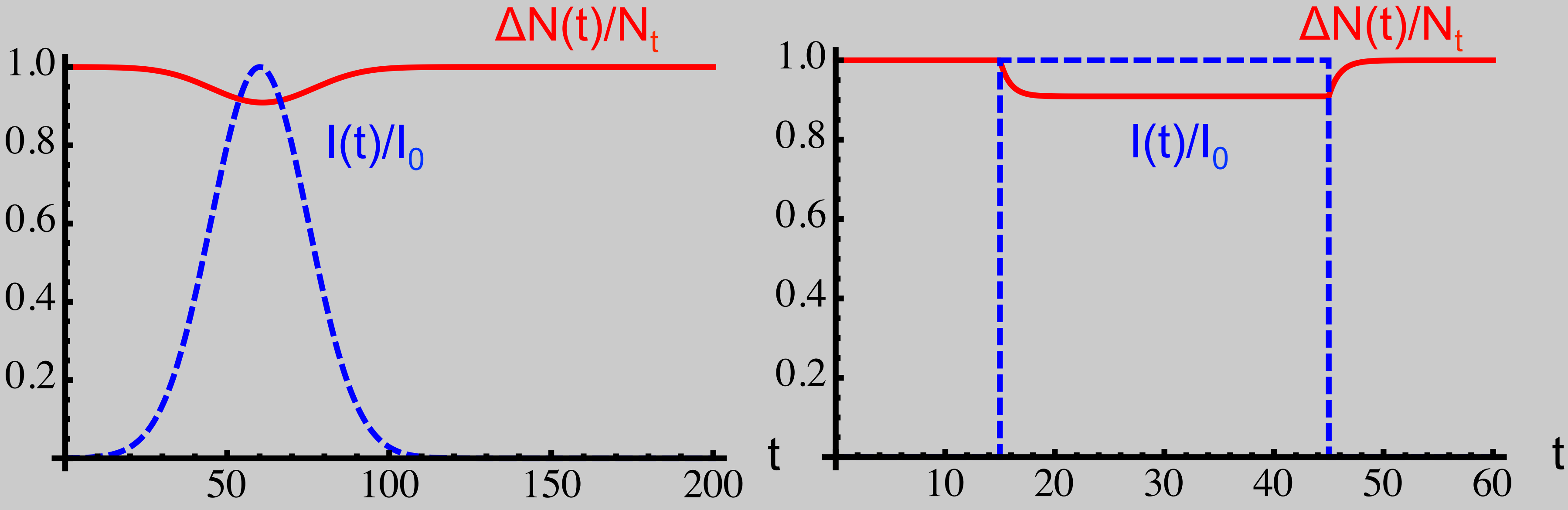
- For *long* pulse input: $\tau_p \gg \tau_{21}$, and peak $I \ll I_{sat}$, $\Delta N(t)$ follows $I(t)$

$$\rightarrow \frac{d}{dt}\Delta N \ll A_{21} N_t$$

Quasi-static, quasi-CW limit

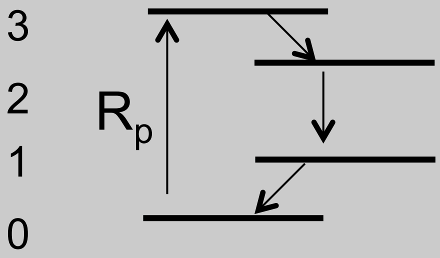
N_t adiabatically follows I(t)

$$\frac{\Delta N}{N_t} = \frac{1}{1 + I(t)/I_{sat}}$$



Gain saturation

- Consider a 4-level system:



Assume: τ_{32} and $\tau_{10} << \tau_{21}$ and $W_{21}N_2$

- Look at level 2 only:

$$\frac{dN_2}{dt} = R_P - W\,N_2 - N_2/\tau_{21}$$

- Steady state:

$$N_2 = \frac{R_P\tau_{21}}{1+W\tau_{21}} = \frac{R_P\tau_{21}}{1+\frac{\sigma_{21}\tau_{21}}{h\nu_{21}}I} = \frac{R_P\tau_{21}}{1+\frac{I}{I_{sat}}}$$

Low intensity: $N_2=R_P\tau_{12}$

τ_{12} is called “storage time”

- Saturation intensity for *gain:*
  - *No factor of 2*

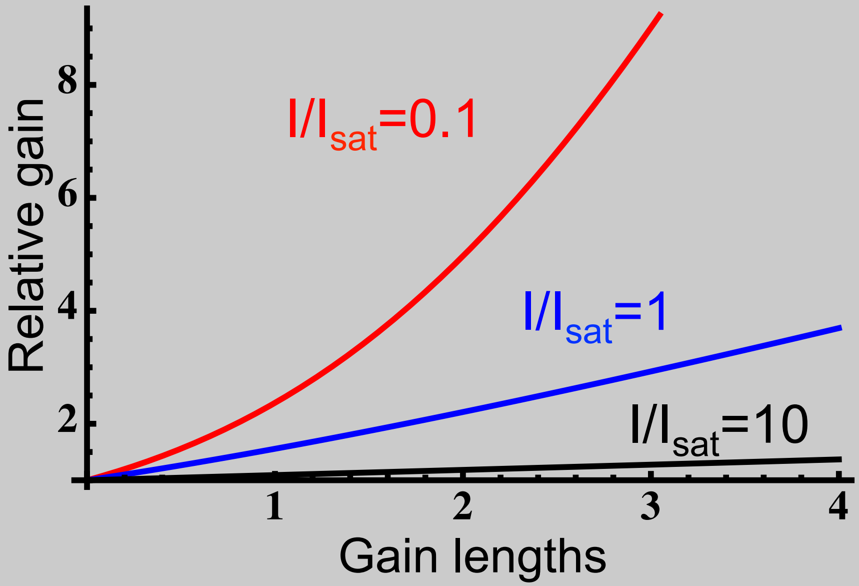
$$\boxed{I_{sat} = \frac{h\nu_{21}}{\sigma_{21}\tau_{21}} = \frac{\Gamma_{sat}}{\tau_{21}}}$$

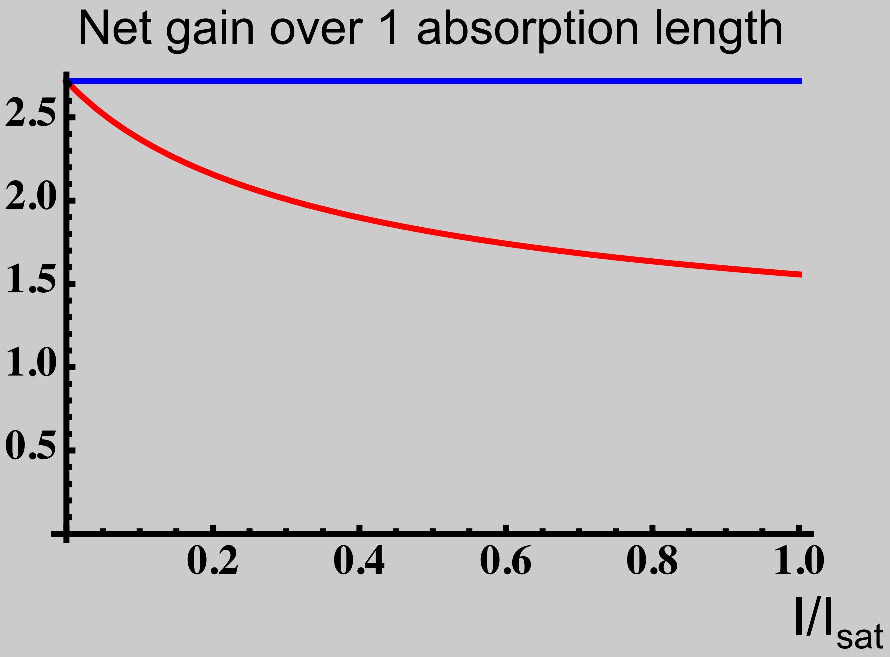
$$\boxed{g(I) = \frac{g_0}{1+I/I_{sat}}}$$

Beam growth during amplification

- Calculation just as with absorption

$$\int_{I_0}^{I}\left(\frac{1}{I}+\frac{1}{I_{sat}}\right)dI=+\int_0^L g_0\,dz\rightarrow\ln\left[\frac{I(z)}{I(0)}\right]+\frac{I(z)-I(0)}{I_{sat}}=+g_0 z$$





- **Even though saturated gain is low, it is efficient at extracting stored energy**

Spatial dependence

- Gain follows distribution of pump intensity
- Spatial variation of gain affects beam profile
- Examples:
  - longitudinal pumping with Gaussian beam leads to gain narrowing of spatial profile. More gain in center, less at edges
  - Saturated absorption by a Gaussian beam: saturation in center suppresses intensity there. Leads to widening of output beam.

Pulse amplification: saturated gain algorithm

Frantz-Nodvick Equation:

$$G = \frac{\Gamma_{sat}}{\Gamma_{seed}} \ln[1+(e^{\Gamma_{seed}/\Gamma_{sat}} - 1)e^{\Gamma_{Pump}/\Gamma_{Sat}}]$$

No Spatial Dependence:

$$F_{Pump} = \frac{E_{Abs}}{\pi w_P^2}$$

$$F_{Seed} = \frac{E_{Seed}}{\pi w_S^2}$$

→ **Frantz-Nodvick** → Gain

Assumptions:

- Thermal Equilibrium within Stark Manifolds
- Square Temporal Profile of Seed

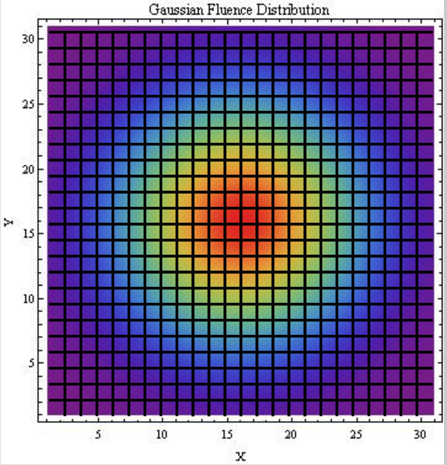
Transverse dependence: super-Gaussian

$$\Gamma(x,y) = \Gamma_0 e^{\frac{-[(\frac{x}{w_x})^{nx}+(\frac{y}{w_y})^{ny}]}{2}}$$

where:- ***nx,ny*** = 2 (Gaussian),
Even > 2 (Super-Gaussian)

-F₀ is defined via the Total Energy and integration of the distribution

$\xrightarrow{(\Delta x, \Delta y)}$



Define Spatial Arrays → **Define Gain Array** *(Frantz-Nodvick for each element in spatial array)* → $F'_{Seed} = G \bullet F_{Seed}$ → **Compute Post-Gain Seed Energy** (Integrate Spatial Array) →

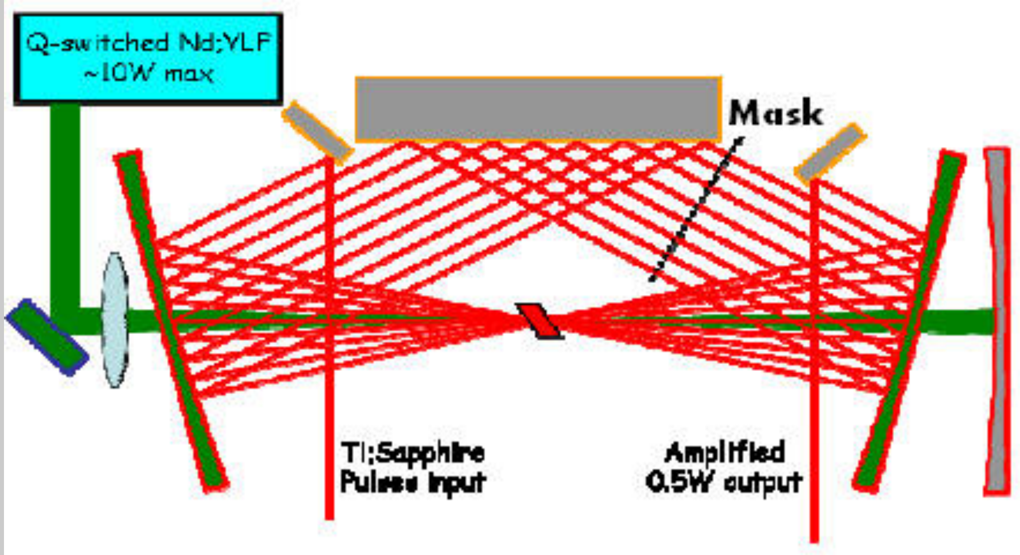
$$Gain = \frac{E'_{Seed}}{E_{Seed}}$$

Example: Ti:sapphire multipass amp

- Seed pulse from pulsed laser oscillator: 1nJ (800nm)
- Amplify to 1mJ, use 7mJ of pump energy (532nm)
- Multipass designs: spatially separate beams

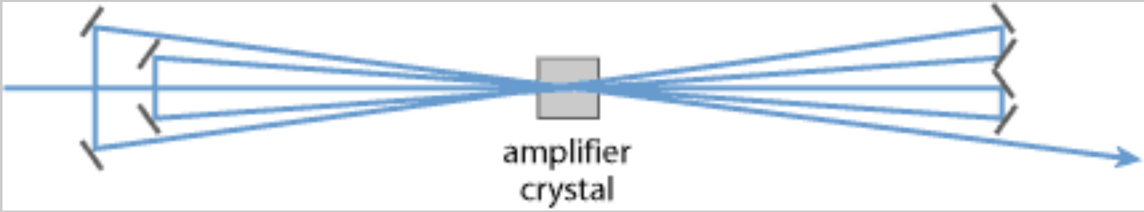
Three-mirror ring preamp:

- Up to 12 passes
- Focused beam in crystal
- 2 mirror alignment



Bowtie power amp:

- Collimated beam
- 8 mirrors



Multipass design

- Assume uniform pumping with round beams
- Calculate stored fluence and small signal gain
- Use saturated gain expression to calculate new energy after 1st pass
- Subtract extracted energy from stored energy (over seed spot area)
- Repeat for N passes

Conditions: 1nJ seed, 7mJ pump energy, 95% absorption, 10% loss/pass

Stored energy:

$$E_{stor} = E_{pump}\eta_{abs}\frac{h\nu_{seed}}{h\nu_{pump}} = 4.4mJ$$

Small signal gain estimate:

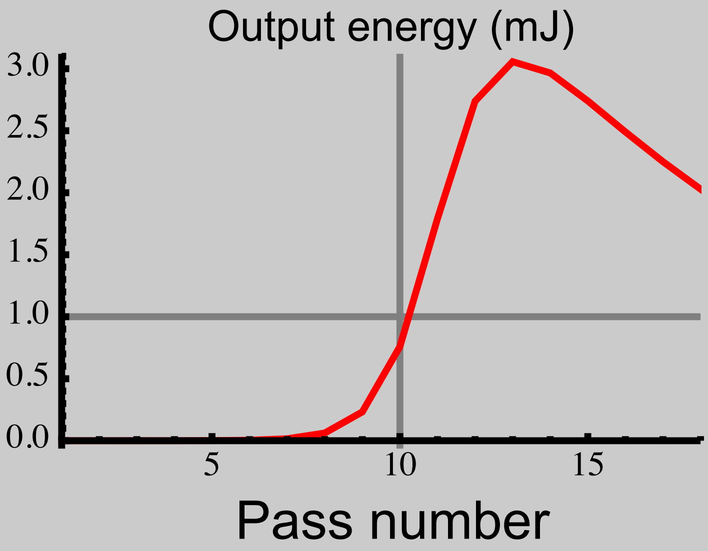
$$G_0 = \left(\frac{E_{\text{target}}}{E_{\text{seed}}}\right)^{1/N}\frac{1}{1-L} = 4.42$$

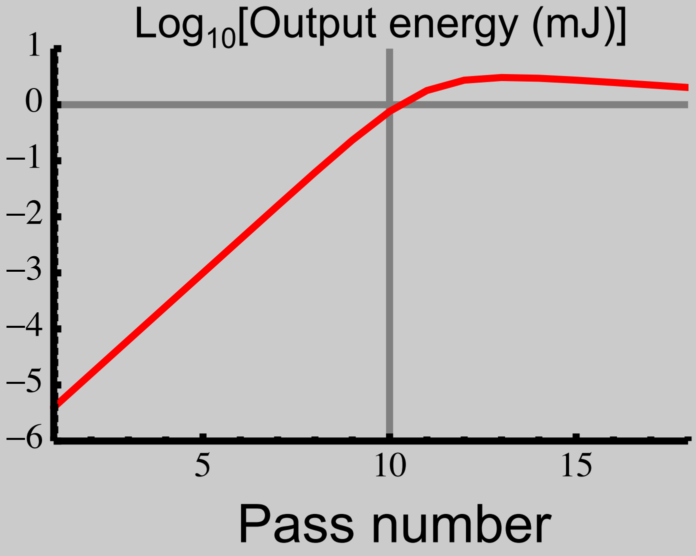
Estimated spot size:

$$A_{pump} = \frac{E_{stor}}{\Gamma_{sat}\ln\left[G_0\right]}, \quad w_p = 300\mu m$$

Multipass: Simple calculated results

- Small signal gain estimate works as long as stored energy is not depleted

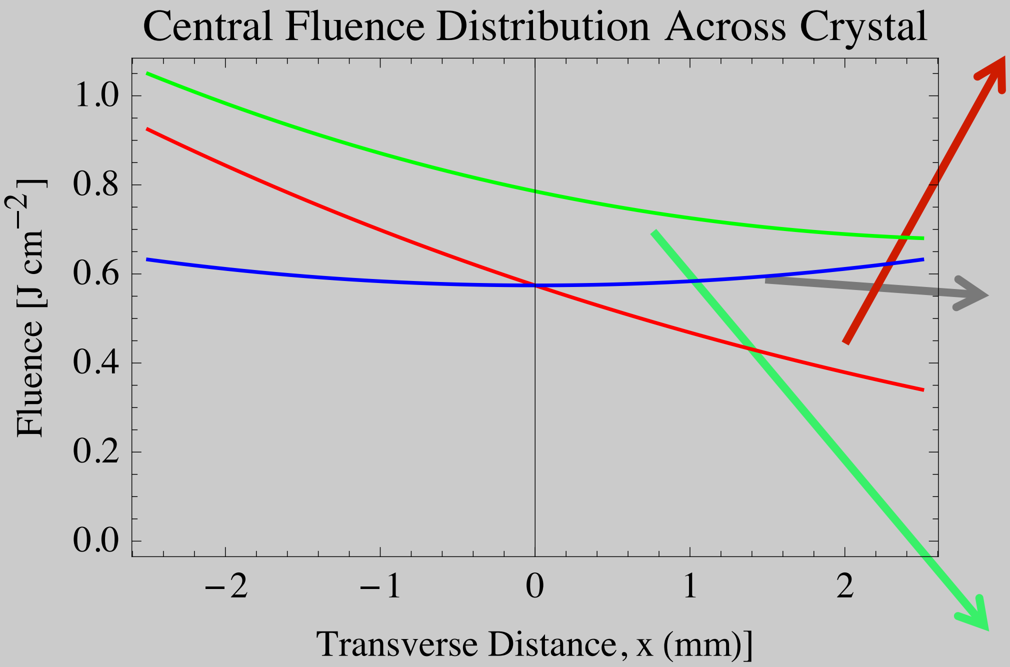


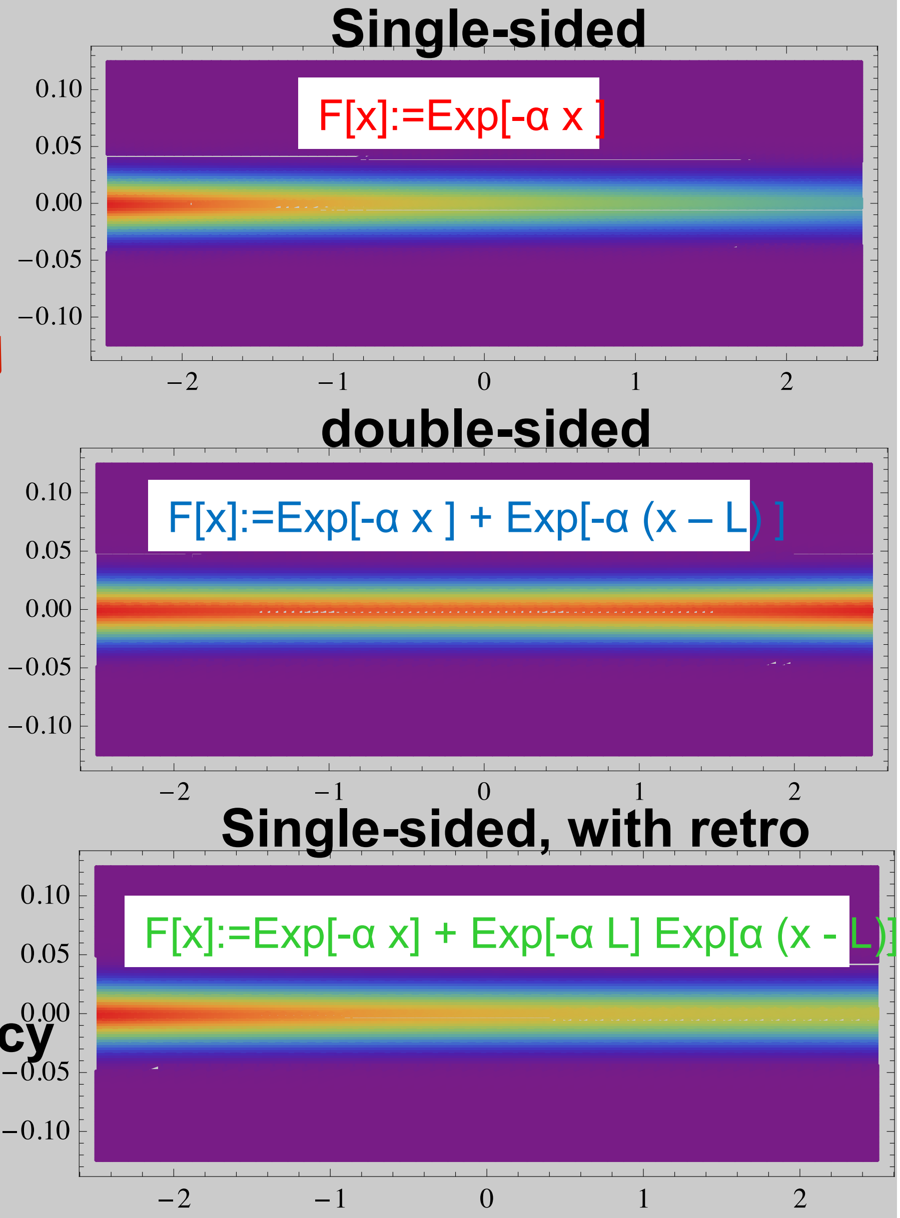


  - Smaller seed size to ensure full overlap with pump
  - Avoid damage thresholds for pump and seed
  - Saturate at desired energy to reduce noise
  - Account for size change in Brewster cut crystal

Transverse diode bar pumping

For good absorption, pump must have sufficient path length





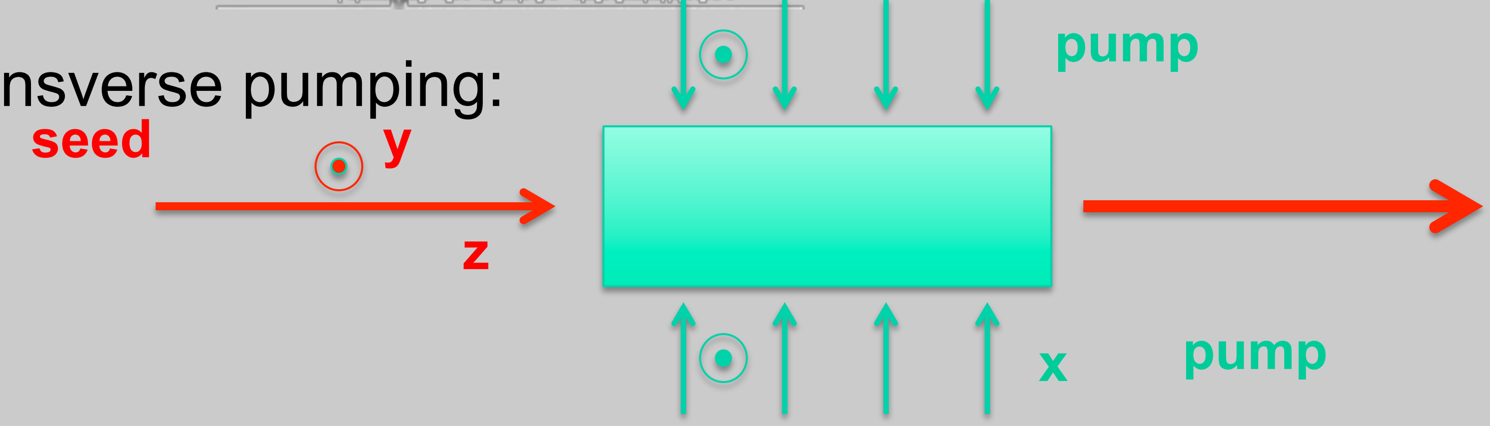
Using retro: better absorption efficiency

Double-sided: better uniformity

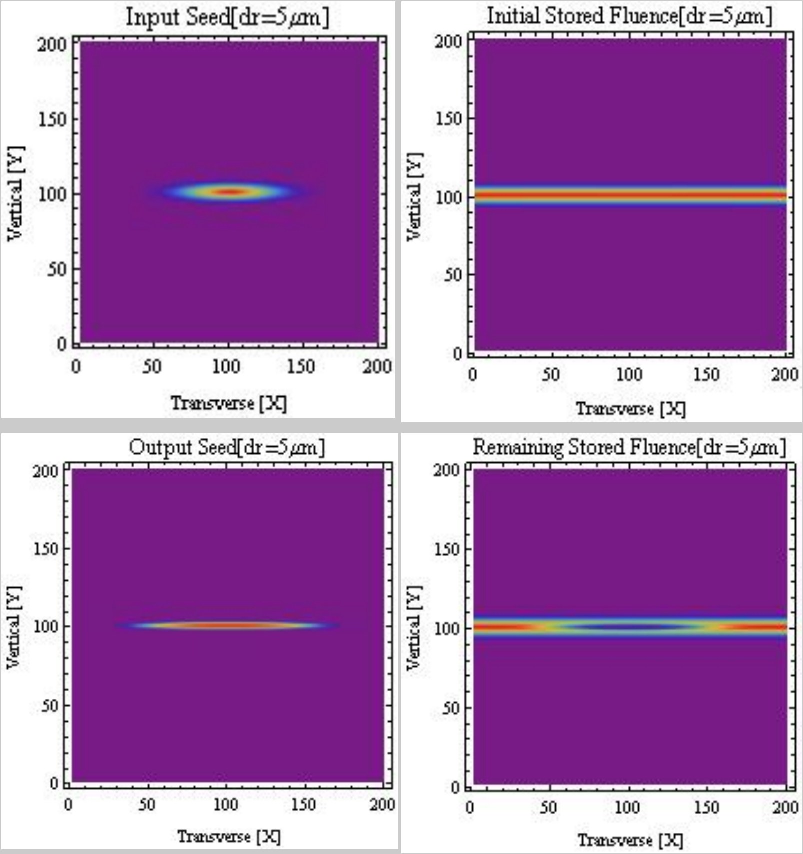
Polarization issues in pumping birefringent materials

- For Ti:sapphire, both polarizations contribute to seed gain along c-axis
- Much higher pump absorption for E along c-axis
  - α across c-axis is about 40% lower than along c-axis

- Ex: transverse pumping:



Transverse Pumping Gain Estimates



- Seed: 2nJ
  - Cavity Losses: ~1%
  - τ_{pass}: 1ns
- Pump (CW): 1kW (Total: 2X .5kW Bars)
  - η_{Abs}=63.2%
  - η_{QD}=55.6%
  - $\eta_{Pump}=\eta_{Abs}\ \eta_{QD}$=35.1%
  - Heat: ~560 W
  - Significant (Cylindrical) Thermal Lens Expected
  - w=30um
- Single Pass Gain (small signal)
  - Astigmatic Seed: g≈1.64
    - w_x=200um, w_y=30μm
  - Spatially Chirped Seed: g≈1.64
    - w_x=2mm, w_y=30μm

•Multi-Pass Extraction: 37 Passes

-Astigmatic Mode: ~136uJ (small extraction area)

-Spatially Chirped:~.53mJ (46% extraction)

Central dip in gain: spatial gain mode *expansion*.
This could be used to counter gain narrowing for spatially-chirped seed

Frequency dependence: account for lineshapes

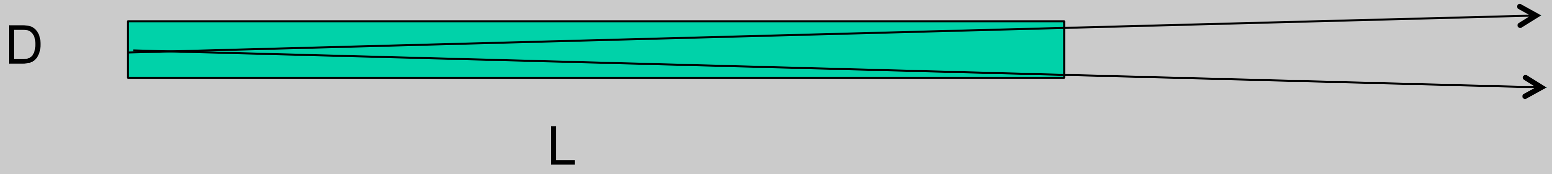
- Absorption and gain coefficients and saturation intensity both depends on frequency

$$\alpha(I,\nu)=\frac{\alpha_0(\nu-\nu_0)}{1+\frac{I(\nu)}{I_{sat}(\nu-\nu_0)}}$$

- For broadband input, saturation changes shape of transmitted spectrum
  - Absorption: power broadening
  - Gain: spectral gain narrowing

Amplified Spontaneous Emission (ASE)

- Spontaneous emission is emitted into 4π steradians, but is amplified on the way out if there is gain.



  - ASE can be considered to be a noise source
  - ASE is more directional than fluorescence, but not as directional as a coherent laser beam
  - Some high-gain lasers are essentially ASE sources (no mirrors)
- Implications for amplifier design
  - ASE can deplete stored energy before pulse extraction
  - Use timing and good seed energy to extract energy from medium before ASE
  - Ensure that transverse gain is smaller than longitudinal to avoid parasitic depletion.

Self-absorption and “optically-thick” media

- A related phenomenon for an absorbing medium is when radiation is *absorbed* along the way out.
- More absorption near the line center, so the transmitted light is broader in spectrum.
- For an extended luminous body (e.g. the Sun), the individual spectral lines get merged together to look like the blackbody.