Today: Hamiltonian stuff

Wed: Finish, central forces

Last time: Pend. in accel. train

$$\ddot{\theta} = -\frac{g}{L}\sin\theta + \frac{a}{L}\cos\theta$$

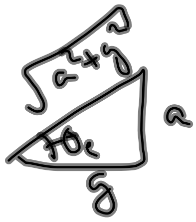
in eq. $\dot{\theta} = \ddot{\theta} = 0 \rightarrow 0 = -\frac{g}{L}\sin\theta_e + \frac{a}{L}\cos\theta_e$

$$\tan\theta_e = \frac{a}{g}.$$

If I say $\theta = \theta_e + \alpha$

$$\ddot{\theta} = -\frac{g}{L}\left(\sin\theta_e\cos\alpha + \cos\theta_e\sin\alpha\right) + \frac{a}{L}\left(\cos\theta_e\cos\alpha - \sin\theta_e\sin\alpha\right)$$

$$\frac{\ddot{\theta}}{\cos\theta_e} = -\frac{g}{L}\left(\cancel{\tan\theta_e\cos\alpha} + \sin\alpha\right) + \frac{a}{L}\left(\cancel{\cos\alpha} - \underbrace{\tan\theta_e}_{a/g}\sin\alpha\right)$$



$$= -\frac{g}{L}\sin\alpha - \frac{a^2}{gL}\sin\alpha$$

$$\ddot{\theta} = -\frac{\cos\theta_e}{gL}\left(a^2+g^2\right)\sin\alpha$$

$$\ddot{\alpha} = (\underbrace{\cancel{\theta_e}}_{\text{const}}+\alpha)^{\cdot\cdot} = \ddot{\alpha} = \ddot{\theta} = -\frac{\sqrt{a^2+g^2}}{L}\sin\alpha$$

Back to theory!

In cartesian coords:

$$T = \frac{1}{2} \sum_{\alpha, i}^{n, i} m_\alpha \dot{x}_{\alpha i}^2$$

n particles. $i = 1, 2, 3$ directions

Transform to gen coordinates:

$$x_{\alpha i} = x_{\alpha i}(q_j, t)$$

$$\dot{x}_{\alpha i} = \sum_{j=1}^{s} \frac{\partial x_{\alpha i}}{\partial q_j} \dot{q}_j + \frac{\partial x_{\alpha i}}{\partial t}$$

$$\dot{x}_{\alpha i}^2 = \sum_{j,k} \frac{\partial x_{\alpha i}}{\partial q_j} \dot{q}_j \frac{\partial x_{\alpha i}}{\partial q_k} \dot{q}_k + 2 \sum_{j} \frac{\partial x_{\alpha i}}{\partial q_j} \dot{q}_j \frac{\partial x_{\alpha i}}{\partial t} + \left(\frac{\partial x_{\alpha i}}{\partial t} \right)^2$$

$$T = \sum_{j,k} a_{jk} \dot{q}_j \dot{q}_k + \sum_j b_j \dot{q}_j + c$$

$$a_{jk} = \frac{\partial x_{\alpha i}}{\partial q_j} \frac{\partial x_{\alpha i}}{\partial q_k} \qquad \left. b_j = \frac{\partial x_{\alpha i}}{\partial q_j} \frac{\partial x_{\alpha i}}{\partial t} \right\} c =$$

If $\frac{\partial x_{\alpha i}}{\partial q_j} = 0$: No explicit t dep in transform

$$\rightarrow \quad T = \sum_{jk} a_{jk} \underbrace{\dot{q}_j \dot{q}_k}_{\text{only quadratic terms in } \dot{q}_j \text{ show up in } T.}$$

$$\frac{\partial T}{\partial \dot{q}_l} = \sum_{jk} \left(a_{jk} \delta_{jl} \dot{q}_k + a_{jk} \dot{q}_j \delta_{kl} \right)$$

$$= \sum_k a_{lk} \dot{q}_k + \sum_j a_{jl} \dot{q}_j$$

$a_{lk} = a_{kl}$ symmetric

$$\sum_l \dot{q}_l \frac{\partial T}{\partial \dot{q}_l} = 2 \sum_{kl} a_{kl} \dot{q}_k \dot{q}_l = 2T$$

$$\boxed{\text{no exp. } t \text{ dep in transform} \rightarrow \sum_l \dot{q}_l \frac{\partial T}{\partial \dot{q}_l} = 2T}$$

$$\frac{dL}{dt} = \underbrace{\sum_j \left(\frac{\partial L}{\partial q_j} \dot{q}_j + \frac{\partial L}{\partial \dot{q}_j} \ddot{q}_j \right)}_{\frac{d}{dt}\left(\dot{q}_j \frac{\partial L}{\partial \dot{q}_j}\right)} + \frac{\partial L}{\partial t} \qquad \underline{\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} = \frac{\partial L}{\partial q_j}}$$

$$\frac{dL}{dt} - \sum_j \frac{d}{dt}\left(\dot{q}_j \frac{\partial L}{\partial \dot{q}_j}\right) = \frac{\partial L}{\partial t}$$

$$\frac{d}{dt}\left(L - \sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_j}\right) = \frac{\partial L}{\partial t}$$

Define

$$H \equiv \sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_j} - L = -\frac{\partial L}{\partial t}$$

$\rightarrow \dfrac{dH}{dt} = -\dfrac{\partial L}{\partial t}$ H is conserved if L doesn't explicitly dep. on time.

If u doesn't depend on $\dot{q}_j$

$$\frac{\partial L}{\partial \dot{q}_j} = \frac{\partial T}{\partial \dot{q}_j} \rightarrow H = \underbrace{\sum_j \dot{q}_j \frac{\partial T}{\partial \dot{q}_j}}_{2T} - T + u = -\frac{\partial L}{\partial t}$$

$$H = T + u$$

H is the total energy if coord. trans. are ind. of time and u does not depend on $\dot{q}_j$.