add $\vec{\nabla}\times f\hat{A} = f\,\vec{\nabla}\times\hat{A} - \hat{A}\times\vec{\nabla}f$

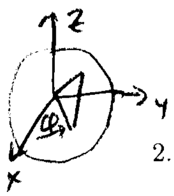
PHGN361 Exam 3: NAME

1. A uniformly charged sphere of mass M and radius R is centered at the origin and rotates at constant angular speed ω_0 about the z axis in a magnetic field $B_1\hat{x}+B_2\hat{z}$. Find an expression for the acceleration of the center of mass.

   Fundamental Principles (4 pts): $\vec{F} = \int \vec{J}\times\vec{B}\,d\tau = M\vec{a}$

   Outline the method of Solution (4 pts):
   Find $\vec{J}$ & $d\tau$, do cross product to get $\vec{F}$ & set $\vec{F}=m\vec{a}$ solve for a

   Check Solution (2 pts): $\omega_0\to 0,\ F\to 0$: $R\to 0,\ F\to 0$: $B\to 0,\ F\to 0$
   Check units

   Complete solution in a form Mathematica will integrate (5 pts): $d\tau = r'^2\sin\theta'\,d\theta'\,d\phi'\,dr'$,

   $$a = \frac{1}{M}\iiint \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ J_x & J_y & \wedge \\ B_1 & 0 & B_2 \end{vmatrix} d\tau$$

   $\vec{J} = \omega_0\times\vec{r} = \omega_0 r'\hat{\phi}\sin\theta$
   $\vec{J} = J_x\hat{x}+J_y\hat{y} = \omega_0 r'(\cos\phi'\hat{x}+\sin\phi'\hat{y})$

2. (15 pts) Show that applying the curl to $\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|}d\tau'$ is consistent with $\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}')\times(\vec{r}-\vec{r}')}{(\vec{r}-\vec{r}')^3}d\tau'$.

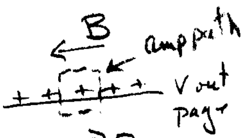
   $\vec{\nabla}\times\vec{A} = \frac{\mu_0}{4\pi}\int \vec{J}(\vec{r}')\times\vec{\nabla}\frac{1}{\mathscr{r}}\,d\tau'$ but $\vec{\nabla}\frac{1}{\mathscr{r}} = \hat{x}\frac{\partial}{\partial x}\frac{1}{[(x-x')^2+\ldots]^{1/2}} + \hat{y}\frac{\partial}{\partial y}\frac{1}{[\ ]^{1/2}}+\ldots$

   & $\vec{\nabla}\times\vec{J}(\vec{r}') = 0$ function of r' ← derivative wrt r

   $\vec{\nabla}\frac{1}{\mathscr{r}} = \hat{x}\frac{2(x-x')}{2[\ ]^{3/2}} + \hat{y}\frac{2(y-y')}{[\ ]^{3/2}} + \cdots = \frac{1}{\mathscr{r}^3}\left((x-x')\hat{x}+(y-y')\hat{y}+\ldots\right) = \frac{\vec{\mathscr{r}}}{\mathscr{r}^3}$ So $\vec{\nabla}\times\vec{A} = \frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r}')\times\vec{\mathscr{r}}}{\mathscr{r}^3}d\tau'$

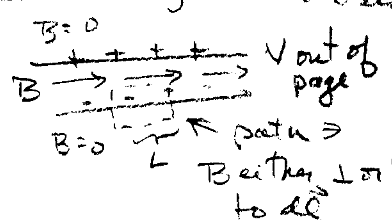
3. A $\boxed{\text{large}}$ parallel-plate capacitor with uniform surface charge σ_0 on the upper plate and $-\sigma_0$ on the lower plate is moving with constant speed V. Find the magnetic field everywhere.

   Fundamental Principles (4 pts): Biot & Savart or Ampere's law & $K=\sigma V$ $\frac{C}{m^2}\times\frac{m}{s}$ $I=KL$
   Find B given current :

   Outline the method of Solution (4 pts):
   Symmetry ⇒ Amperes Law. Need to find path & I enclosed then $\oint\vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$
   OR find B for one plate & use superposition

   Superpose soln with neg plate

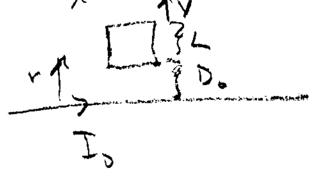
   

   Check Solution (2 pts): $V\to 0$ $B\to 0$; $\sigma_0\to 0,\ B\to 0$
   check bndry condition $B_{\parallel}^{above}-B_{\parallel}^{below} = \mu_0 K$

   Complete integrated solution (5 pts): $\oint\vec{B}\cdot d\vec{l} = BL = \mu_0 KL$

   $B = \mu_0 K = \mu_0\sigma_0 V$ inside and $B=0$ outside

4. (15 pts) At some instant, a square wire with each side of length L is located a distance D_0 from an infinite wire carrying constant current I_0. If, at this instant, the square wire moves at speed V away from the infinite wire find the emf (if any) in the square wire.

   Fund. Principle : Faraday's Law $Emf = -\frac{d\Phi_m}{dt}$

   Method: find B (Amps Law) find da & $\Phi_m = \int\vec{B}\cdot d\vec{a}$

   $B = \frac{\mu_0 I_0}{2\pi r}$ $da = L\,dr$ $\Phi_m = \int_D^{D+L}\frac{\mu_0 I_0 L\,dr}{2\pi r} = \frac{\mu_0 I_0 L}{2\pi}\ln\left(\frac{D+L}{D}\right)$

   $Emf = -\frac{d\Phi_m}{dt}$ but what depends on time? Since square loop is moving away $D(t) = D_0+Vt$

   $Emf = -\frac{d}{dt}\left\{\frac{\mu_0 I_0 L}{2\pi}\ln\left[\frac{D_0+Vt+L}{D_0+Vt}\right]\right\}$