Laser transients

Relaxation oscillations

return to 4-level rate equations

$$\frac{dN}{dt} = \underset{\text{pump}}{R_p} - \underset{\text{stim. em.}}{B\phi N} - \underset{\text{spont em.}}{\frac{1}{\tau} N} \qquad B = \frac{\sigma c}{V_{cav}} = \frac{\sigma}{A_{beam}} \frac{c}{L_{cav}}$$

$$\frac{d\phi}{dt} = \left(\underbrace{B V_a N}_{\text{stim. emiss.}} - \frac{1}{\tau_c}\right)\phi$$

↳ cavity decay

photon lifetime: $\tau_c = \frac{L_{eff}}{\gamma c}$ $\qquad \gamma = -\ln(t)$

Equilibrium values

$N_{th} = \frac{\gamma}{\sigma l}$ threshold inv. density = eqm. N

$R_{pc} = N_{th}/\tau$

$\Phi_0 = \frac{A_b}{\sigma} \gamma \frac{\tau_c}{\tau} \left(\frac{R_p}{R_{pc}} - 1 \right)$ eqm photon num.

Start with $N = 0$ $\quad \phi = 1$ (one photon in cavity mode)

turn on const. pump $R_p > R_{pc}$

→ spiking, gradual change to damped oscillation.

timescales:

1) pump phase: no lasing $\phi \approx 0$

$$\frac{dN}{dt} = R_p - \frac{1}{\tau} N$$

2) lasing: Fast build up of ϕ R_p, $1/\tau \approx 0$

$$\frac{dN}{dt} \approx -B\phi N$$

$$\frac{d\phi}{dt} = \left(BV_aN - \frac{1}{\tau_c}\right)\phi$$

→ pulse: rapid build up until gain is depleted.
terminates when $N < N_{th}$.

3) re-pumping
- ϕ at higher value at start of next cycle.

4) oscillations:
consider small departure from equilibrium

$$N = N_0 + \delta N$$

$$\phi = \phi_0 + \delta\phi$$

insert to linearize NL equations:

$$\frac{d}{dt}\delta N = R_p - B(\phi_0 + \delta\phi)(N_0 + \delta N) - \frac{1}{\tau}(N_0 + \delta N)$$

- drop product terms $\delta\phi\,\delta N$
- equilibrium values → 0

$$\frac{d}{dt}\delta N = -B(\phi_0\delta N + N_0\delta\phi) - \frac{1}{\tau}\delta N$$

$$\frac{d}{dt}\delta\phi = BV_a(N_0 + \delta N)(\phi_0 + \delta\phi) - \frac{1}{\tau_c}(\phi_0 + \delta\phi)$$

$$= \underbrace{\left(BV_aN_0 - \frac{1}{\tau_c}\right)}_{}(\phi_0 + \delta\phi) + BV_a\phi_0\,\delta N$$

$$\frac{d^2}{dt^2}\delta\phi \overset{=0}{=} BV_a\phi_0\frac{d}{dt}\delta N = BV_a\phi_0\left(-\left(B\phi_0 + \frac{1}{\tau}\right)\delta N - BN_0\delta\phi\right)$$

$$\frac{d^2}{dt^2}\delta\phi + \cancel{BV_a\phi_0}\left(B\phi_0 + \frac{1}{\tau}\right)\left(\frac{1}{\cancel{BV_a\phi_0}}\right)\frac{d}{dt}\delta\phi + B^2 V_a N_0 \phi_0 \delta\phi = 0$$

define characteristic rates:

$$\frac{2}{t_0} = B\phi_0 + \frac{1}{\tau}$$

$$\omega^2 = B^2 V_a N_0 \phi_0$$

$$\rightarrow \frac{d^2}{dt^2}\delta\phi + \frac{2}{t_0}\frac{d}{dt}\delta\phi + \omega^2\delta\phi = 0$$

expect exponential solution: $\delta\phi_0 e^{pt}$

$$\rightarrow p^2 + \frac{2}{t_0}p + \omega^2 = 0$$

solutions:

$$p = -\frac{1}{t_0} \pm \sqrt{\frac{1}{t_0^2} - \omega^2}$$

if $\omega^2 > 1/t_0 \rightarrow$ im in root $\quad \omega' = \sqrt{\omega^2 - 1/t_0^2}$

$$\delta\phi(t) = \delta\phi_0 e^{-t/t_0}\sin(\omega' t + \beta)$$

damped oscillations

$\delta N(t)$ is out of phase.

Inv. density and intracavity energy oscillate.

$\rightarrow$ "Relaxation oscillations"

Can see how feedback - eg. back reflections. will affect laser stability.

$t_0 \sim$ damping time

$$\frac{1}{t_0} = \frac{1}{2}\underbrace{\frac{\sigma c}{V_{cav}}}_{B}\frac{A_b}{\sigma}\gamma\frac{\tau_c}{\tau}(x-1) + \frac{1}{\tau} \qquad x = \frac{R_p}{R_{pc}} = \frac{P}{P_{th}}$$

$$\frac{\tau}{t_0} = \frac{1}{2}\,\frac{c\gamma}{L_{eff}}\,\frac{L_{eff}}{\gamma_c}\,(x-1) + 1 = \frac{1}{2}(x-1) + 1 = \frac{1}{2}(x+1)$$

$$t_0 = \frac{2\tau}{x+1}$$ damping comes from fluor. decay, - faster if pumping above thresh.

$$\omega^2 = B^2 V_a N_0 \Phi_0$$

$$= \left(\frac{\sigma c}{V_{cav}}\right)^2 V_a \,\frac{\gamma}{\sigma l}\,\frac{A_b}{\sigma}\,\gamma\,\frac{\tau_c}{\tau}(x-1)$$

$$= \frac{c^2 l}{L_{eff}} \cdot \frac{1}{L_{eff}}\,\frac{\gamma^2}{l}\,\frac{L_{eff}}{\gamma_c}\,\frac{1}{\tau}(x-1)$$

$$= \frac{x-1}{\tau_c \tau}$$ osc period btw τ, τ_c - shorter when pumping $\gg P_{th}$.

If $t_0 < 1/\omega$, $\sqrt{\ } \rightarrow$ real, no oscillations.

damping time is fast

$$t_0 \omega < 1$$

$$= \frac{2\tau}{x+1}\sqrt{\frac{x-1}{\tau_c \tau}} < 1$$

$$\frac{x-1}{\tau_c \tau} < \frac{(x+1)^2}{4\tau^2} \rightarrow \frac{\tau}{\tau_c} > \frac{(x+1)^2}{4(x-1)}$$

$$\frac{\tau_c}{\tau} > \frac{4(x-1)}{(x+1)^2}$$

In this regime (fast fluor time, i.e. not much storage)
→ only damping, no spiking

Other instabilities:

- longitudinal mode beating
  - competition for gain when standing waves overlap.
- spatial mode hopping.
- noise seeds relaxation oscillations.
  - > pump noise, vibrations
- chaotic behavior
- account for bandwidth, other non-ideal rates ($\tau_{32}, \tau_{10} \ldots$)