

Review:

$$\text{Curl } \vec{E} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$$\vec{\nabla} \times \vec{G} = -\frac{\partial \vec{B}}{\partial t}$$

$$\text{divergence } \vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \cdot \vec{G} = 0$$

G is the Faraday field.

$$\text{Let } \vec{E}_{tot} = \vec{E} + \vec{G}$$

$$\vec{\nabla} \cdot \vec{E}_{tot} = \vec{\nabla} \cdot (\vec{E} + \vec{G}) = \vec{\nabla} \cdot \vec{E} + \vec{\nabla} \cdot \vec{G} = \rho / \epsilon_0$$

$$\vec{\nabla} \times \vec{E}_{tot} = \vec{\nabla} \times (\vec{E} + \vec{G}) = \vec{\nabla} \times \vec{E} + \vec{\nabla} \times \vec{G} = -\frac{\partial \vec{B}}{\partial t}$$

Since the G field generates a force on a charge that is the same as experienced by an E we rewrite $G=E$

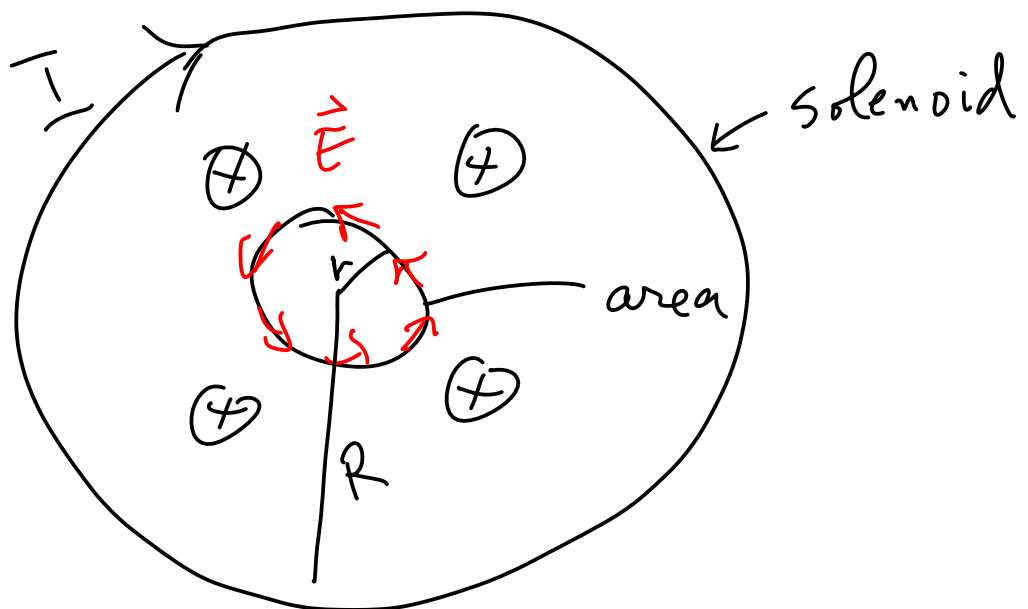
$$\vec{F} = q(\vec{E} + \vec{G}) + q\vec{v} \times \vec{B}$$

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\int \vec{\nabla} \times \vec{E} \cdot d\vec{a} = - \frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a} = - \frac{\partial \Phi}{\partial t}$$

$$\oint \vec{E} \cdot d\vec{r} = - \frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a}$$



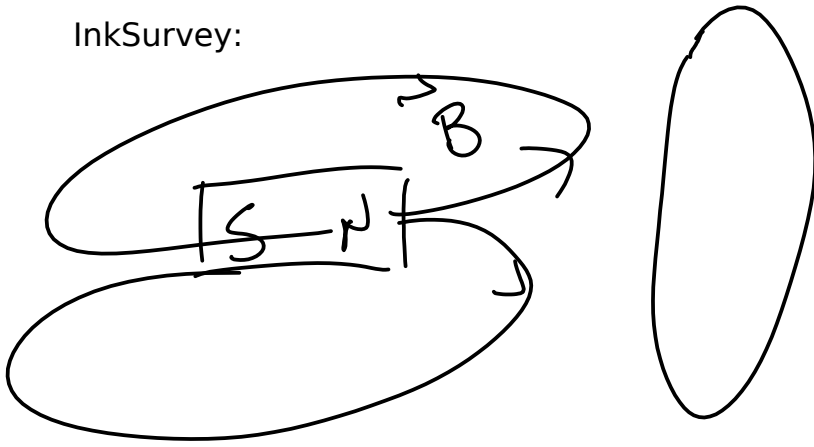
$$E 2\pi r = - \frac{\partial}{\partial t} B \pi r^2 = - \pi r^2 \frac{\partial B}{\partial t}$$

$$\vec{E} = - \frac{r}{2} \frac{\partial B}{\partial t} \hat{\phi}$$

$$B = \mu_0 n I(t)$$



InkSurvey:



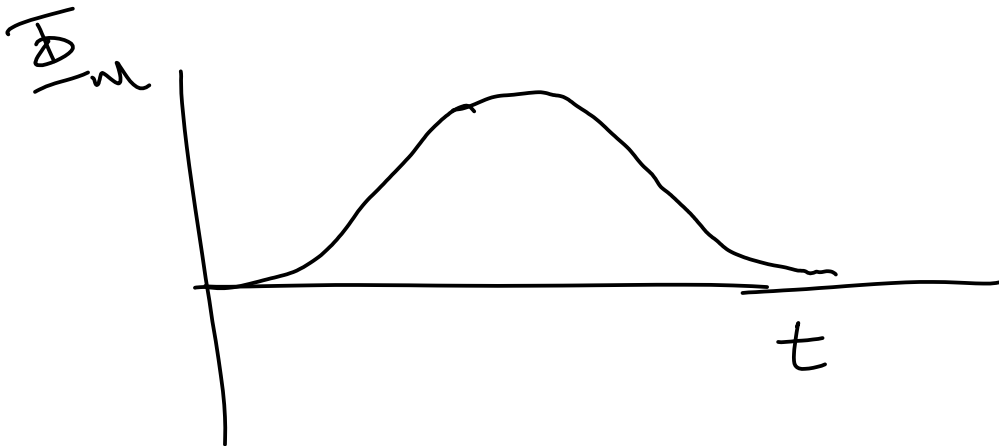
Knee-jerk thinking

no definitions or
fundamental eqns used

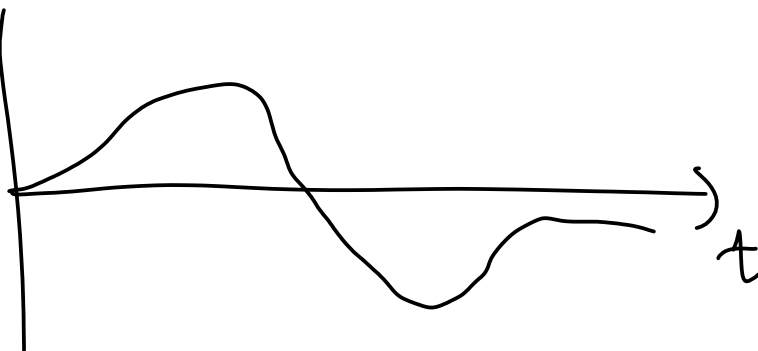
Analytic thinking

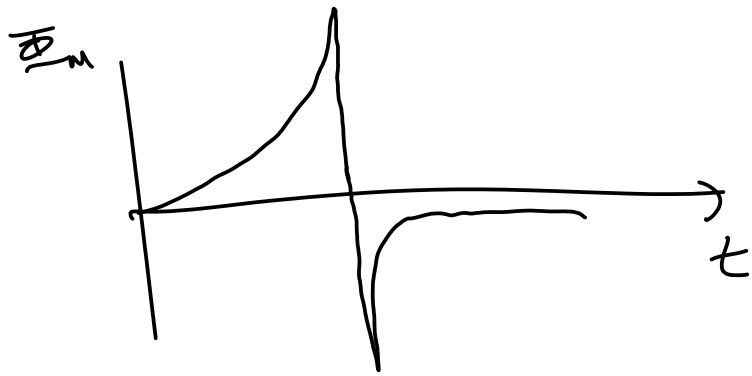
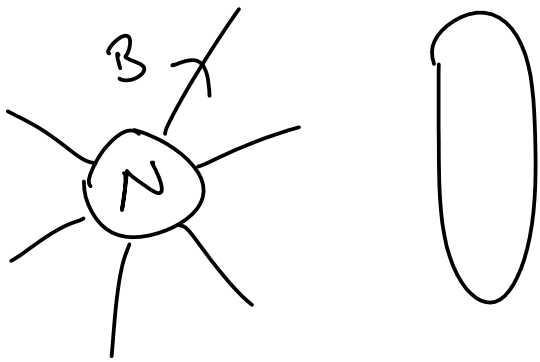
$\oint \vec{E} \cdot d\vec{r}$ generates
current

$$\left| \oint \vec{E} \cdot d\vec{r} \right| = \frac{d\Phi_m}{dt}$$

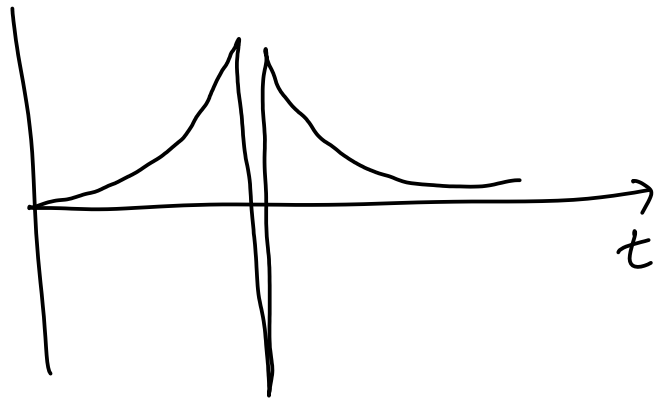


$I \propto \frac{d\Phi_m}{dt}$





$$I \propto \frac{d\Phi_m}{dt}$$



Muddiest points

- How do you use Faraday's law at a point?

Integral form

$$\oint \vec{E} \cdot d\vec{r} = - \frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a}$$

Differential form

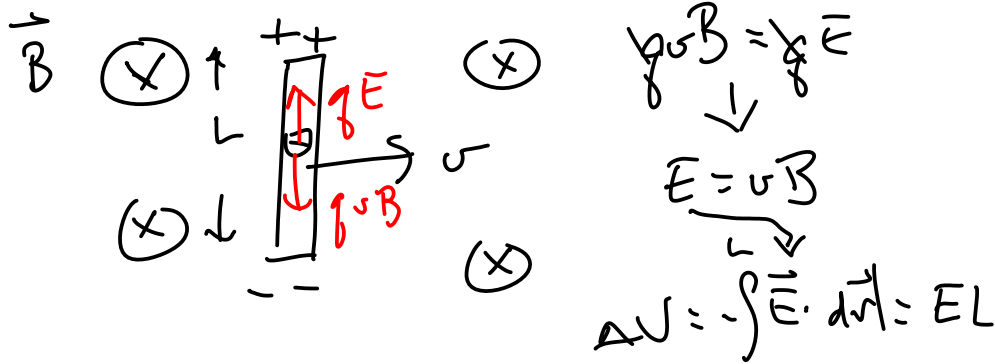
$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

Given E near a fixed point its curl is the time rate of change of B at that point.

What does the partial derivative mean?

It means to fix all other variables. In Faraday's law x, y, z are fixed while time varies. That is, we are not moving in x, y, z while using Faraday's. We are fixed in position.

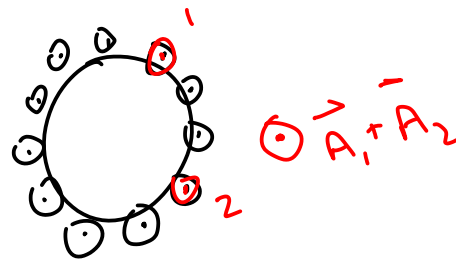
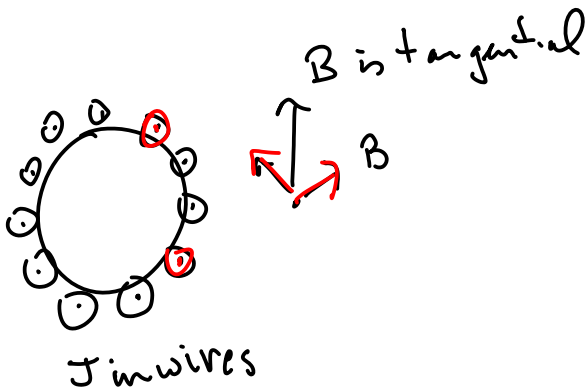
How do you calculate the voltage if the rod accelerates through a B ?



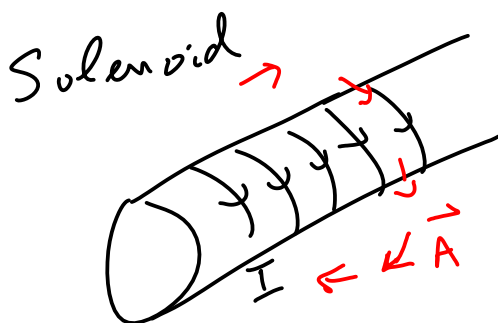
We have not determined how to work with accelerating charges. The "quasistatic approximation means that charges are moving slowly enough to neglect the effects of acceleration. Then just replace v by a .

How do you determine the direction of A given J ?

For symmetric current distributions A and J are in the same direction. This is like using Biot-Savart to find B knowing currents in a symmetrical situation.



A from wire with surface current



Questions about hmwk will be addressed in the posted solns.

Is it possible to create a constant E with a B?

For the solenoid problem above just make sure $\frac{\partial B}{\partial t} = \text{constant}$

$$B = \alpha t \rightarrow \infty \text{ as } t \rightarrow \infty$$

Other ways to measure A? Should I find A from B?

The math to determine radiation will start with A and V NOT E and B. We measure radiation consistent with this and not consistent when starting with E and B.

Still confused about the skydiver analogy.

$$J = \rho v \left\{ \begin{array}{l} \text{skydivers} \\ \text{charges} \end{array} \right.$$

$$\vec{J} = \left\{ \begin{array}{l} \sigma \vec{g} \quad \text{skydivers} \\ \sigma \propto \text{air pressure} \\ \sigma \vec{E} \quad \text{charges} \\ \sigma \text{ material} \end{array} \right.$$

small σ large σ for constant ρ

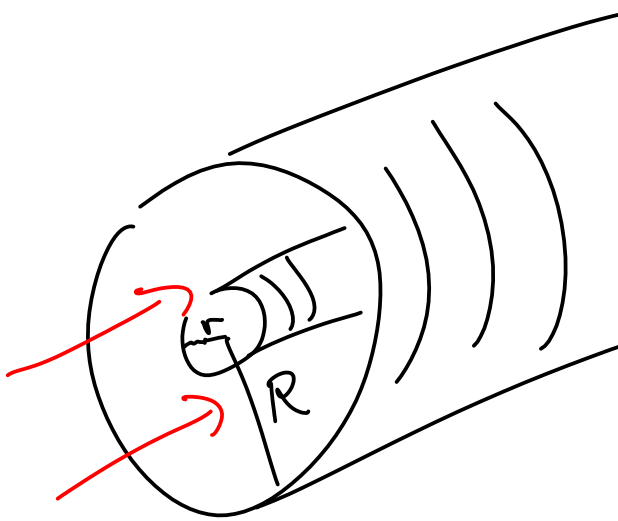
Changing B generates a current which creates another B. Where does this process end?

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

What B is this?

This is the TOTAL B from all currents.

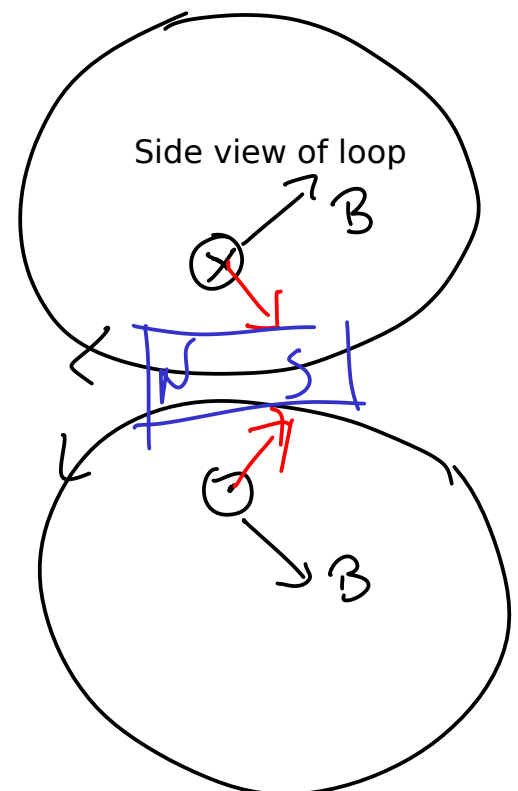
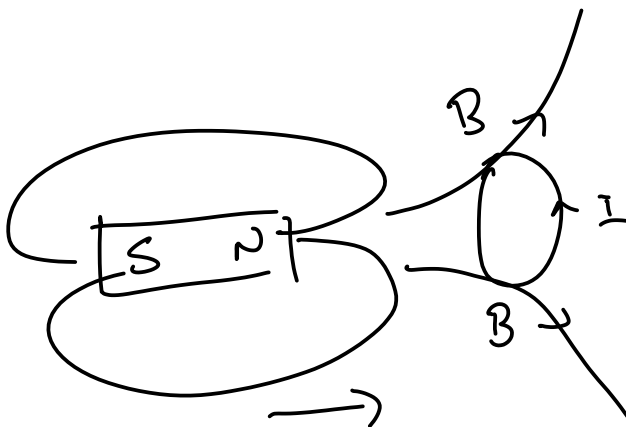
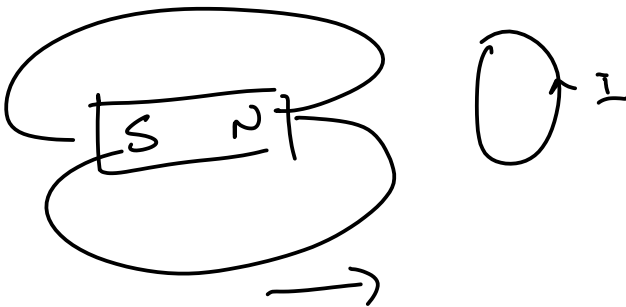
$$B_R = \mu_0 n_R I(t)$$



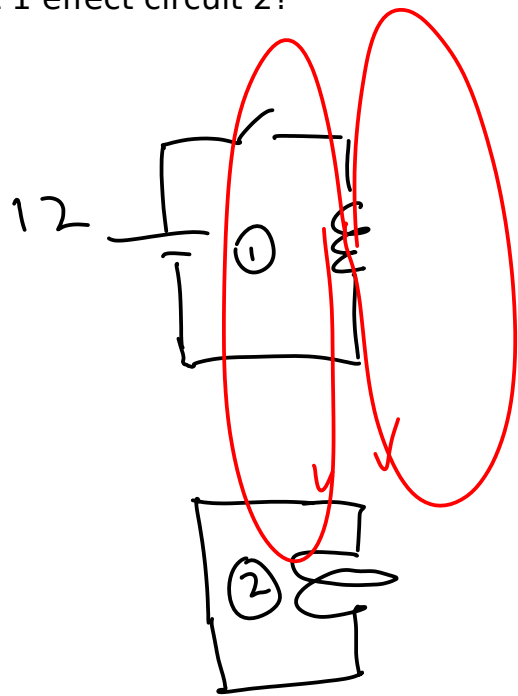
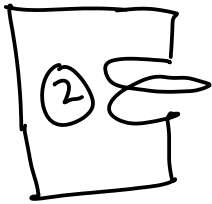
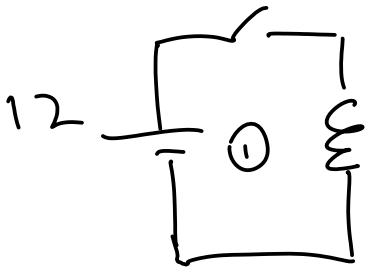
The outer solenoid generates a changing flux in the inner solenoid causing a current to flow. The inner solenoid then generates an extra changing B which causes an extra E both in the inner and outer solenoids. This extra E then affects the currents in the solenoids etc.

The minus sign in Faraday's law is called Lenz's law.

The electric field is generated in a direction which will attempt to cause a current to flow which opposes the changing flux.



How does circuit 1 effect circuit 2?



(causal/creative) What fundamental law governs this behavior?

$$\frac{\Phi_m^{2 \text{ due } 1}}{\Phi_m} \propto ?$$

$$\frac{\Phi_m^{2 \text{ due } 1}}{\Phi_m} \propto I_1$$

$$\frac{\Phi_m^{2 \text{ due } 1}}{\Phi_m} \equiv M_{21} I_1$$

defn of M

$$\oint \vec{E}_2 \cdot d\vec{r} = M_{21} \frac{dI_1}{dt}$$

assuming M_{21} constant

mutual inductance

$$\oint \vec{E}_2 \cdot d\vec{r} \text{ around one loop} = \Delta V \text{ or } \mathcal{E} \text{mf}$$

$$\text{For } N \text{ loops in circuit 2, } \Delta V_{\text{tot}} = \Delta V_1 + \Delta V_2 + \dots \\ = N \Delta V$$

For each loop there is an Emf. However this is taken into account in the defn of the mutual inductance via the flux. That is,

$$\Phi_{\text{tot}}^{\text{2 due 1}} = N \Phi_{\text{one loop}} \equiv M_{21} I_1$$

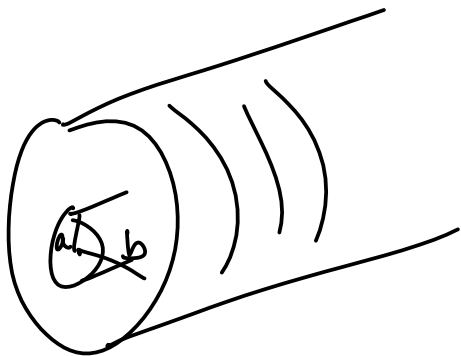
(analogous) What analogous relation is there in electrostatics?

$$M = \frac{\Phi_m}{I} \quad \frac{\text{flux}}{\text{amp}} \qquad C = \frac{Q}{V} \quad \frac{\text{Coul}}{\text{volt}}$$

(congruous) How do I calculate M?

Use the defn. by putting current I_1 in one circuit and finding the flux through the other.

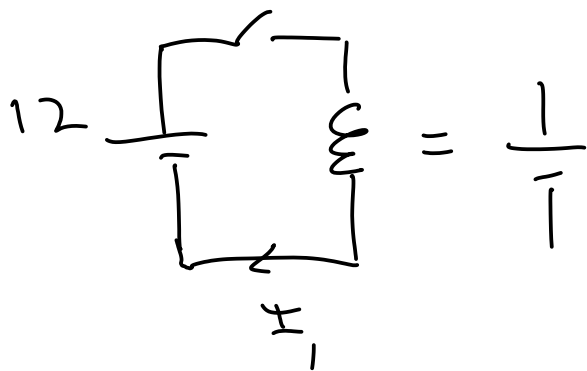
The standard example is one solenoid of radius a inside a larger solenoid of radius b .



$$N_a \vec{B} \pi a^2 = \Phi_{\text{tot}} = M_{21} I_1 \\ \uparrow \\ \text{due to solenoid b} \\ \text{"} \\ \mu_0 n_b I_1$$

note that the I_1 's cancel

$$N_a \mu_0 n_b \pi a^2 = M_{21} \\ n_a L$$



$$\Phi_1 \propto I_1$$

Questions about this model?

-(congruous) Why does this have a non-zero curl?

-(congruous) What there be a time dependence in E?

-(congruous) What time dep of A.

-(congruous) what does the minus sign signify?

-(cong) continuity eqn

-(cong) What does Stokes theorem tell us?

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

Since this a non-conservative field maybe we should call it G rather than E. Remember the curl of E is zero.

$$\nabla \times \vec{A} = - \frac{\partial \vec{A}}{\partial t}$$

$$\int \nabla \times \vec{E} \cdot d\vec{a} = - \frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a} = - \frac{\partial}{\partial t} \Phi$$

$$\int \vec{E} \cdot d\vec{r} = - \frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a}$$

$$\mathcal{E}_{\text{ind}} = - \frac{d\Phi_1}{dt} = -L \frac{dI_1}{dt}$$