Take home part of exam 1 PH361

1. This problem involves a solution of the heat diffusion equation $((\kappa \nabla^2 T = c_v \partial T / \partial t c_v)$ using separation of variables. It is an extension of the problem on assignment 6. Take a spherical material (say a potato) and assume it is of radius 1 meter (grown in Idaho) and the temperature is fixed forever to be zero at this boundary (winter time in Idaho). If you had any questions about your solution to this problem in assignment 6 go to

http://www.astro.cf.ac.uk/undergrad/module/PX3104/tp4/node14.html

Now, by some unknown means (a microwave pulse, for example, is focused into the potato) we generate a temperature distribution in the potato given by $T(r, t = 0) = \exp[-a(r-b)^2]/r$. Choose a = 10,000, b = 0.1, $c_v = 1$, and $\kappa = 1$. You are to solve this problem with 100 terms in the separation of variables series expansion using Mathematica.

(a)Plot the initial temperature distribution as a function of r along with the separation of variables solution at t = 0.

(b)Generate a table of snapshots in time with time going from 0 to 0.001 in increments of 0.0001 of the temperature distribution in the potato.

(c)Why does the change in temperature with time and with radius make sense in terms of the diffusion equation.