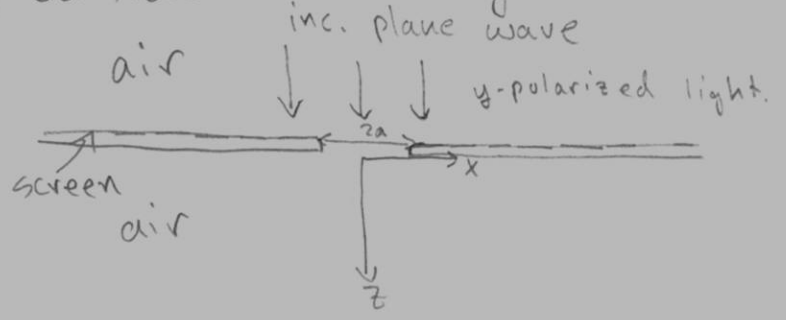
9) Consider the single slit situation shown below:



use the Kirchoff integral from wikipedia (Fresnel Diffraction article), which I also wrote in the lecture to set up an integral for the field $E_y(x,z)$ for $z>0$. Assume

$$E_y(x,z=0) = \begin{cases} 0, & |x|>a \\ E_0, & |x|<a. \end{cases}$$

a) Write down the integral in its analytical form.

b) Make a function in Mathematica

Ey[x_, z_, a_, λ_, E0_] := NIntegrate[<Integral from (a)>]

c) Setting $E_0=1$, $\lambda = 580\text{nm}$, $a=\lambda/4$, plot on the same axis as a function of x, E_y for $z=\lambda, 5\lambda, 10\lambda, 50\lambda, 100\lambda$. Plot over the range $-10\lambda < x < 10\lambda$.

d) On the same set of axes, plot E_y as a function of x at $z=50\lambda$ for

$a=\lambda/4, \lambda/2, \lambda, 5\lambda, 10\lambda, 20\lambda$.

Plot over the range $-30\lambda < x < 30\lambda$.

10) For the waveguide in problem 8, instead of free space wavelength $\lambda_0 = 1550\text{nm}$, use $\lambda_0 = 580\text{nm}$.

a) How many bound modes are there now?

b) Plot the normalized mode profiles for each mode on the same set of axes. Note that $x = \emptyset$ is at the center of the waveguide.

c) Using an incident field $E_0(x) = 1$, find $\langle E_{b,i} | E_0 \rangle$ for all the bound modes $E_{b,i}$.

d) Using an incident field $E_0(x) = x e^{-x^2/b^2}$, fin $\langle E_{b,i} | E_0 \rangle$ for each bound mode.