Impulse response

δ-function input $\quad \delta(t)$ = ultrashort pulse

$\delta(x,y)$ = point source

$h(t) = \hat{\mathcal{L}}\, \delta(t)$ = impulse response

$h(t-t_0) = \hat{\mathcal{L}}\, \delta(t-t_0)$ same fcn $h()$ if shift invariant

impulse response contains all info on LSI systems.

Transfer functions

suppose $f(t) = e^{-i\omega_0 t}$ input a pure frequency

$$g(t) = \hat{\mathcal{L}}\, f(t) = A e^{i\phi} e^{-i\omega_0 t}$$

$$= H(\omega_0)\, f(t)$$

$H(\omega_0) = A e^{i\phi}$ a complex number

in words $e^{-i\omega_0 t}$ = eigenfunction

$H(\omega_0)$ = eigenvalue (complex)

$A(\omega_0)$ = amplitude factor (gain or loss)

$\phi(\omega_0)$ = phase shift

A, ϕ are real functions

output is the same ω_0, but ampl. and phase are changed.

$H(\omega) \equiv$ transfer function

Characterization of linear systems.

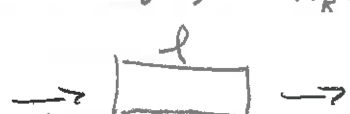
1) directly measure transfer fcn $H(\omega)$
   source - tune ω_{in}
   measure amplitude $A(\omega)$
   measure phase shift $\phi(\omega)$ (need a way)

   or put in all ω's at once (white light, noise)

2) measure impulse response $h(t)$
   short pulse input
   measure $a(t)$, $\Phi(t)$

FT or FT^{-1} to get results in other domain.

Physical example:
complex refractive index
$$n(\omega) = n_R(\omega) + i\, n_I(\omega)$$



$$e^{-i\omega t} \qquad e^{-i\omega t}\, e^{ik\ell} \qquad k = \frac{\omega}{c} n(\omega)$$

$$e^{-i\omega t} \underbrace{e^{i\frac{\omega}{c} n_R(\omega)\ell}\; e^{-\frac{\omega}{c} n_I(\omega)\ell}}_{H(\omega)}$$

phase shift, $\phi(\omega) = \frac{\omega}{c} n_R(\omega)\ell$

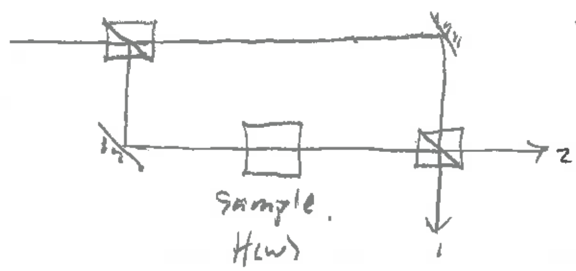
$A(\omega) = e^{-\frac{\omega}{c} n_I(\omega)\ell}$

$\Big\}\; H(\omega) = A e^{i\phi}$

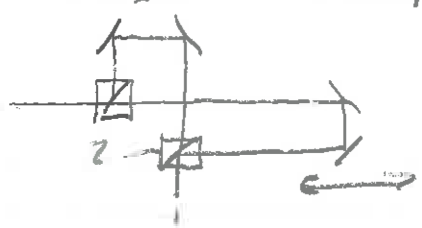
absorption if $n_I(\omega) > 0$
gain if < 0

Spectral interferometry
- measure frequency response.

set up Mach-Zehnder interferometer.
- avoids feed back to laser.
- access to both ports.



want to adjust time delay?



translation stage $\tau = (L_2 - L_1)/c$

without sample $g(t) = f(t) + f(t-\tau)$

$$G(\omega) = F(\omega) + F(\omega) e^{i\omega\tau}$$
$$|G(\omega)|^2 = 2|F(\omega)|^2 (1 + \cos\omega\tau)$$

with sample $\quad G(\omega) = F(\omega)(1 + H(\omega) e^{i\omega\tau})$

measure intensity $\quad I_{int}(\omega) = |G(\omega)|^2 = |F(\omega)|^2 \left(1 + |H(\omega)|^2 + H(\omega) e^{i\omega\tau} + H^*(\omega) e^{-i\omega\tau}\right)$

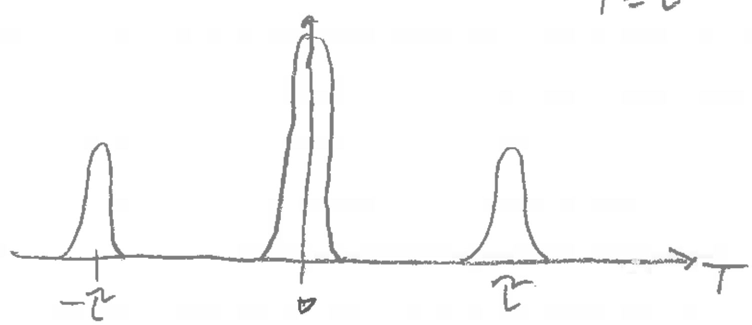
consider the FT^{-1} of this spectral intensity

$$I_{int}(T) = \frac{1}{2\pi} \int_{-\infty}^{\infty} I_{int}(\omega) e^{-i\omega T} d\omega$$

with no sample:

$$I_{int}(\omega) = 2|F(\omega)|^2 + |F(\omega)|^2 e^{i\omega\tau} + |F(\omega)|^2 e^{-i\omega\tau}$$

D.C. + sidelobe at $T=\tau$ − sidelobe at $T=-\tau$



all same shape.

with sample:

$$I_{int}(\omega) = |F(\omega)|^2(1+|H(\omega)|^2) + H(\omega)|F(\omega)|^2 e^{i\omega\tau} + c.c.$$

DC + −

complex $H(\omega)$ is encoded in the sidelobes!
amplitude + phase.

procedure: $FT^{-1}(I(\omega))$
mask to select side lobe:
$\mathrm{rect}\left(\frac{T-\tau}{\tau/2}\right) FT^{-1}(I(\omega))$
recenter to remove shift
FT back to ω space:
$\rightarrow \frac{\tau}{2}\,\mathrm{sinc}\left(\frac{T}{\tau}\right) \otimes \left(H(\omega)|F(\omega)|^2\right)$

obtain $H(\omega)$ with resolution $\sim \pi/\tau$