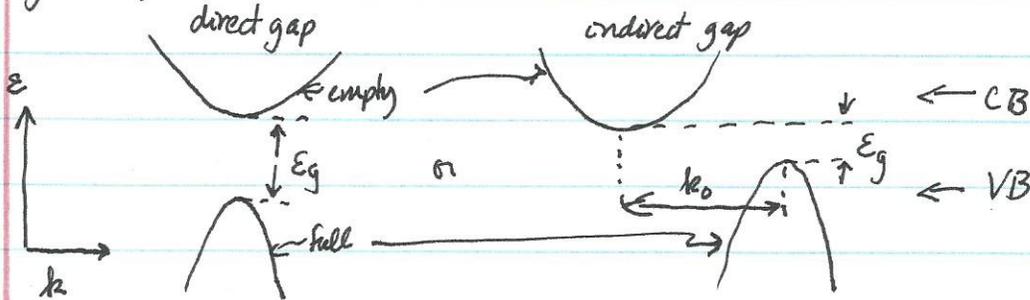


## Semiconductors

In semiconductors and insulators, the ground state is such that there is an energy gap for excitations, i.e., a certain minimum energy (in the thermodynamic limit) is required for absorption of energy. In general, at  $T=0$ :



$$E = E_c + \frac{\hbar^2 k^2}{2m_e^*}$$

$$E = E_c + \frac{\hbar^2 k^2}{2m_e^*}$$

$$E = E_v - \frac{\hbar^2 k^2}{2m_h^*}$$

$$E = E_v - \frac{\hbar^2 (k-k_0)^2}{2m_h^*}$$

For simplicity, consider only direct gap:  
 Si:  $E_g \sim 1.12 \text{ eV}$   
 Ge:  $E_g \sim 0.67 \text{ eV}$

At finite  $T$ , particle and hole excitations exist:  $e$ 's in CB,  $h$ 's in VB

$$\frac{N_{el}}{V} = n_{el} = \frac{1}{V} \int_{CB} f(E) \quad \frac{N_h}{V} = n_h = \frac{1}{V} \int_{VB} (1-f(E))$$

$$n_{el} = \frac{1}{V} \int_{E_c}^{\infty} dE g_c(E) \frac{1}{e^{(E-\mu)/kT} + 1} \quad n_h = \frac{1}{V} \int_{E_v}^{+\infty} dE g_v(E) \frac{1}{e^{-(E-\mu)/kT} + 1}$$

$$= \frac{1}{V} g_c(1) \tau^{3/2} \int_0^{\infty} dx \frac{x^{1/2}}{e^{x-\eta} + 1} \quad (\eta \equiv \frac{\mu - E_c}{kT}) \quad n_h = \frac{1}{V} g_v(1) \tau^{3/2} \int_0^{\infty} dy \frac{y^{1/2}}{e^{y-\xi} + 1}$$

$$g_{c,v}(1) = \frac{V}{2\pi^2} \left( \frac{2m_{e,h}^*}{\hbar^2} \right)^{3/2}$$

$$\xi \equiv \frac{\mu - E_v}{kT}$$

Non-degenerate (classical) regime

$$\bar{e}^{\eta} \gg 1, \bar{e}^{\xi} \gg 1$$

or  $\frac{e^{-(\mu - E_c)/\tau} \gg 1}{e^{(\mu - E_v)/\tau} \gg 1} \approx e^{(\mu - E_c)/\tau} \ll 1, e^{(E_v - \mu)/\tau} \ll 1$  (1)

( $\mu$  at least a few  $\tau$  below  $E_c$ , at least a few  $\tau$  above  $E_v$ )

$$n_{e1} \approx \frac{1}{V} \mathcal{D}_c(\eta) \tau^{3/2} \int_0^{\infty} dx x^{1/2} \bar{e}^{-x} e^{\eta} \equiv n_c e^{\eta} = n_c e^{(\mu - E_c)/\tau}$$

$$n_c \equiv 2 \left( \frac{m_e^* \tau}{2\pi\hbar^2} \right)^{3/2} = e \text{ density if } \mu = E_c \text{ (extreme)}$$

$$n_h \approx \frac{1}{V} \mathcal{D}_v(\xi) \tau^{3/2} \int_0^{\infty} dy y^{1/2} \bar{e}^{-y} e^{\xi} \equiv n_v e^{\xi} = n_v e^{(E_v - \mu)/\tau}$$

$$n_v \equiv 2 \left( \frac{m_h^* \tau}{2\pi\hbar^2} \right)^{3/2} = h \text{ density if } \mu = E_v \text{ (extreme)}$$

(1)  $n_e e^{-\mu/\tau} = n_c e^{-E_c/\tau}$        $n_e n_h = n_c n_v e^{-E_g/\tau} \equiv n_i^2$        $n_e < n_c$   
 $n_h e^{\mu/\tau} = n_v e^{E_v/\tau}$        $n_h < n_v$

Intrinsic:  $n_e = n_h = n_i, \mu \equiv \mu_i$       (1)+(2)  $\Rightarrow$   $\frac{n_e}{n_c} \ll 1 \Rightarrow$  non-degen  
 $\frac{n_h}{n_v} \ll 1 \Rightarrow$  "

(2)  $n_i e^{-\mu_i/\tau} = n_c e^{-E_c/\tau}$        $e^{2\mu_i/\tau} = \frac{n_v}{n_c} e^{(E_v + E_c)/\tau} = \left( \frac{m_h^*}{m_e^*} \right)^{3/2} e^{(E_v + E_c)/\tau}$   
 $n_i e^{\mu_i/\tau} = n_v e^{E_v/\tau}$        $\mu_i = \frac{1}{2}(E_v + E_c) + \frac{3}{4}\tau \ln\left(\frac{m_h^*}{m_e^*}\right)$

General (extrinsic)  $n_e - n_h = \Delta n = n_d^+ - n_a^-$

from (1):  $n_{e1} = n_c e^{(\mu - \mu_i)/\tau}$        $2n_i \sinh\left(\frac{\mu - \mu_i}{\tau}\right) = \Delta n$  (4)  
 and (2):  $n_h = n_v e^{-(\mu - \mu_i)/\tau}$       (3)

$\mu > \mu_i \Leftrightarrow \Delta n > 0$       "n" carriers are majority  
 $\mu < \mu_i \Leftrightarrow \Delta n < 0$       "p" carriers are majority

$$n_e n_h = n_e (n_e - \Delta n) = n_i^2$$

from (3):  $\Rightarrow n_e = \frac{\Delta n}{2} + \sqrt{\left(\frac{\Delta n}{2}\right)^2 + n_i^2}$

$$\mu - \mu_i = \tau \ln\left(\frac{n_e}{n_i}\right) = \tau \ln\left(\frac{\Delta n}{2n_i} + \sqrt{\left(\frac{\Delta n}{2n_i}\right)^2 + 1}\right)$$

$$\mu = \mu_i + \tau \ln\left(\frac{\Delta n}{2n_i} + \sqrt{\left(\frac{\Delta n}{2n_i}\right)^2 + 1}\right) \quad (5)$$

## Impure Semiconductors

### Donors (P in Si, As in Ge)



$\tau=0$ :

extra electron, remains in bound hydrogen-like orbit

$$\text{at } E_d = E_c - \frac{m_e^*}{m} 13.6 \text{ eV}$$

$$a_0 \rightarrow \frac{m_e}{m} a_0 \sim 100 a_0, \frac{m_e^*}{m} \sim 0.1 \quad \epsilon \sim 10 \rightarrow 13.6 \text{ eV}$$

$\tau=0$ :  $\mu > E_d$  extra  $e^-$  remains bound - donor not ionized

$\rightarrow 13.6 \text{ meV}$

### Acceptors (Al in Si, Ga in Ge)



$\tau=0$ :

Ga binds an  $e^-$  from valence band of Ge, hole remains in bound

hydrogen-like orbit at

$$E_a = E_v + \frac{m_h^*}{m} 13.6 \text{ eV}$$

$\tau=0$ :  $\mu < E_a$  hole remains bound - acceptor not ionized.

Because these impurity orbitals are within <sup>few</sup>  $\sim 1 \text{ meV}$  of the conduction and valence band edges, for  $\tau \sim E_c - E_d, E_a - E_v$ , impurities will ionize, and the charges freed into CB and VB can conduct current:  $\tau \sim 10 \text{ meV} \Rightarrow T \sim 100 \text{ K}$

[ Pure semiconductors are hard to make, but can make compensated

semiconductors with  $N_d^+ = N_a^-$ , which will behave as pure, since

$$n_h = n_e - \Delta n \quad (\Delta n = n_d^+ - n_a^-) ]$$

## Donors

(e.g. P in Si, As in Ge)



Statistics of donors: (Chap. 5)

(empty) ionized donor:  $E=0$   $N=0$  1 state must excite el from  $E_d$  to 0

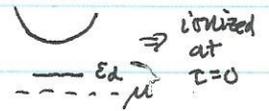
neutral donor:  $E=E_d$   $N=1$  2 states (spin)



$$g = 1 + 2e^{-(E_d - \mu)/\tau}$$

ionized state is non-degenerate

$$P_d(\text{ionized}) = \frac{1}{1 + 2e^{-(E_d - \mu)/\tau}}$$



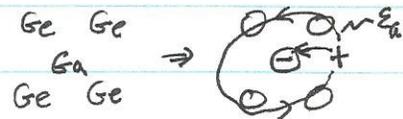
$$\frac{N_d}{V} = \frac{N_d^+}{V} + \frac{N_d^0}{V} = \frac{N_d}{V} P_d(\text{ionized}) + \frac{N_d}{V} (1 - P_d(\text{ionized}))$$

$$\therefore \text{density of ionized donors} \equiv n_d^+ = \frac{N_d}{V} P_d(\text{ionized}) = \frac{n_d}{1 + 2e^{-(E_d - \mu)/\tau}}$$

$$n_d^+ \approx n_d \text{ for } (E_d - \mu) \gg \tau$$

## Acceptors

(e.g. Al in Si, Ga in Ge)



Statistics of acceptors:

ionized acceptor:  $E=E_a$   $N=1$  1 state (must excite an el from VB to  $E_a$  to fill localized hole state)

(empty (no holes)) neutral acceptor:  $E=0$   $N=0$  2 states (spin)



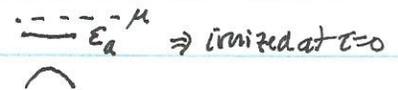
$$g = 2 + e^{-(E_a - \mu)/\tau}$$

$$P_a(\text{ionized}) = \frac{e^{-(E_a - \mu)/\tau}}{2 + e^{-(E_a - \mu)/\tau}} = \frac{1}{1 + 2e^{+(E_a - \mu)/\tau}}$$

$$\frac{N_a}{V} = n_a$$

$$n_a^- = \frac{n_a}{1 + 2e^{-(E_a - \mu)/\tau}}$$

$$n_a^- \approx n_a \text{ for } (\mu - E_a) \gg \tau$$



We can now find  $\mu$  from neutrality condition:

$$n_{el} + n_a^- = n_h + n_d^+$$

p-n junction equilibrium

n type

$$\left. \begin{aligned} n_c \gg n_e \text{ (non-degen.)} \\ \Delta n \approx n_d \gg n_i \end{aligned} \right\} n_e \approx \Delta n \approx n_d \quad n_h \approx \frac{n_i^2}{\Delta n} = \left(\frac{n_i}{n_d}\right)n_i \ll n_i$$

$\therefore n_c \gg n_d \gg n_i$

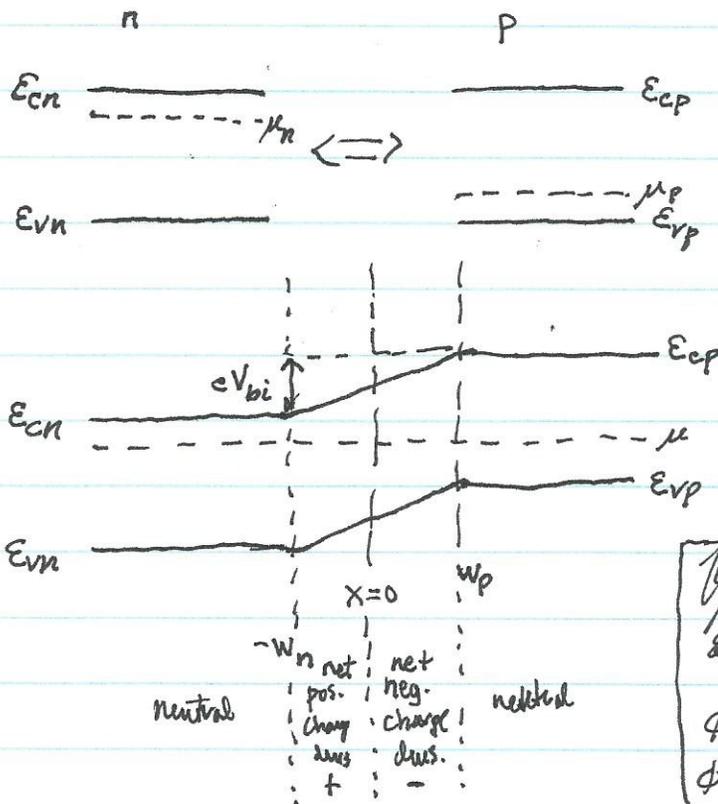
p type

$$\left. \begin{aligned} n_v \gg n_h \text{ non-degen} \\ \Delta n = -n_a \Rightarrow |\Delta n| \approx +n_a \gg n_i \end{aligned} \right\} n_h \approx |\Delta n| \approx n_a \quad n_e \approx \frac{n_i^2}{|\Delta n|} = \frac{n_i}{n_a} n_i \ll n_i$$

$\therefore n_v \gg n_h \gg n_i$

n: all donors ionized  $\Rightarrow E_d - \mu \gg \tau \Rightarrow E_c - \mu \gg \tau \Rightarrow$  non-degen.

p: all acceptors ionized  $\Rightarrow \mu - E_a \gg \tau \Rightarrow \mu - E_v \gg \tau \Rightarrow$  non-degen.



$V_{bi} \equiv$  "built in" potential

$$\begin{aligned} E_c(x) &= E_{cn} - e\phi(x) \\ E_v(x) &= E_{vp} - eV_{bi} - e\phi(x) \\ \phi(-\infty) &= 0 \\ \phi(+\infty) &= -V_{bi} \end{aligned}$$

Forward bias

Finding  $V_{bi}$ :

$$n_e = n_c e^{(\mu - E_c)/\tau}$$

$$n_h = n_v e^{(E_v - \mu)/\tau}$$

In a static potential,  $\phi(x)$ :  $E_c \rightarrow E_c - e\phi(x)$

$$E_v \rightarrow E_v - e\phi(x)$$

since all electron energies are shifted by a static potential in this way.

$$n_e(-\infty) = n_d = n_c e^{(\mu - E_c + e\phi(-\infty))/\tau}$$

$$n_h(+\infty) = n_a = n_v e^{(E_v - e\phi(+\infty) - \mu)/\tau}$$

Then

$$\frac{n_d n_a}{n_c n_v} = e^{-\left(E_g + e(\phi(+\infty) - \phi(-\infty))\right)/\tau}$$

$$\text{or } e(\phi(+\infty) - \phi(-\infty)) = -E_g - \tau \ln\left(\frac{n_d n_a}{n_c n_v}\right) \equiv -eV_{bi}$$

$$\text{So } eV_{bi} = E_g + \tau \ln\left(\frac{n_d n_a}{n_c n_v}\right) \quad \therefore eV_{bi} \sim E_g!$$

Qualitative description of p n junction in equilibrium

1. n type: has excess free - charges  
p type: has excess free + charges

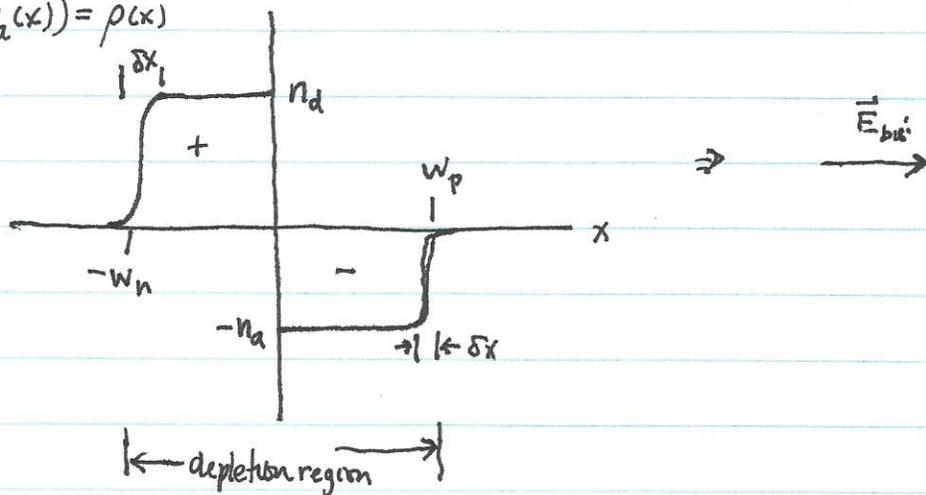
2. Bring n, p together, free charges attract, flow (- to p, + to n) until E field builds up enough to block flow ( $\mu$  same in n, p)

3. Built-in E field in region called "depletion region" (depleted of free charge  $n_c, n_v$ ) because any free charge left ~~would~~ <sup>has</sup> been swept out by built-in E

4. In depletion region, free +, - have "recombined" (annihilated) leaving only fixed charge:  $n_d$  <sup>ionized</sup> donors,  $n_a$  <sup>ionized</sup> acceptors. These fixed charges are what gives the E field in depletion region:

$$-\frac{d^2\phi}{dx^2} = \frac{\rho(x)}{\epsilon} = \frac{e(n_d(x) - n_a(x))}{\epsilon}$$

$$e(n_d(x) - n_a(x)) = \rho(x)$$



$$\delta x \approx \frac{\epsilon}{E_g} (w_n + w_p) \ll (w_n + w_p)$$

↑  
 $eV_{bi}$

Finding the size of the depletion region

$$\frac{d^2\phi}{dx^2} = -\frac{\rho(x)}{\epsilon} = \begin{cases} -\frac{en_d}{\epsilon} & -W_n \leq x < 0 \\ \frac{en_a}{\epsilon} & 0 \leq x \leq W_p \\ 0 & \text{otherwise} \end{cases}$$

$$\phi(x=0^+) = \phi(x=0^-) \quad \left(\frac{d\phi}{dx}\right)_{0^+} = \left(\frac{d\phi}{dx}\right)_{0^-}$$

$$\phi(x) = \begin{cases} \phi(-\infty) & x < -W_n \\ \phi(-\infty) - \frac{en_d}{2\epsilon}(x+W_n)^2 & -W_n \leq x \leq 0 \\ \phi(\infty) + \frac{en_a}{2\epsilon}(x-W_p)^2 & 0 \leq x \leq W_p \\ \phi(\infty) & W_p < x \end{cases}$$

$$\phi(0^+) = \phi(0^-)$$

$$\phi(-\infty) - \frac{en_d}{2\epsilon}W_n^2 = \phi(\infty) + \frac{en_a}{2\epsilon}W_p^2 \Rightarrow \frac{2\epsilon}{e}(\phi(-\infty) - \phi(\infty)) = \frac{2\epsilon}{e}V_{bi}$$

$$\left(\frac{d\phi}{dx}\right)_{0^+} = \left(\frac{d\phi}{dx}\right)_{0^-}$$

$$= n_d W_n^2 + n_a W_p^2$$

$$-\frac{en_d W_n}{\epsilon} = -\frac{en_a W_p}{\epsilon} \Rightarrow n_d W_n = n_a W_p$$

solve :

$$W_n = \sqrt{\frac{n_a}{n_d} \frac{2\epsilon V_{bi}}{e(n_a + n_d)}}$$

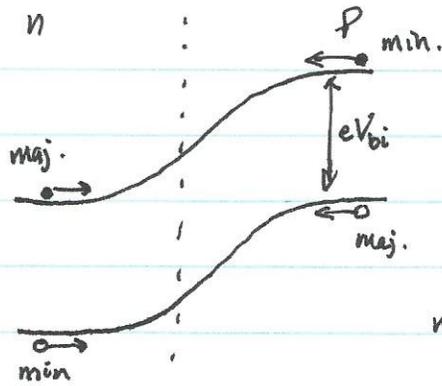
$$W_p = \sqrt{\frac{n_d}{n_a} \frac{2\epsilon V_{bi}}{e(n_a + n_d)}}$$

$$W_n + W_p \propto \sqrt{V_{bi}}$$

Apply potential  $V$  :  $V_{bi} \rightarrow V_{bi} - V$

$V > 0 \Rightarrow$  smaller depletion region ("forward bias")

$V < 0 \Rightarrow$  larger depletion region ("reverse bias")



equilib.:

$$J_e(p \rightarrow n) \equiv J_e^{gen}$$

$$J_e(n \rightarrow p) = \phi_e e^{-eV_{bi}/\tau} = J_e^{gen}$$

↑  
barrier activation probability

non-equilibrium:

$$V_{bi} \rightarrow V_{bi} - V$$

$$J_e(n \rightarrow p) = \phi_e e^{-eV_{bi}/\tau} e^{eV/\tau} = J_e^{gen} e^{eV/\tau}$$

~~J\_e^{gen}~~  $J_e^{gen}$  unchanged from its equilib. value

$$\therefore J_e = J_e(n \rightarrow p) - J_e(p \rightarrow n) = J_e^{gen} (e^{eV/\tau} - 1)$$

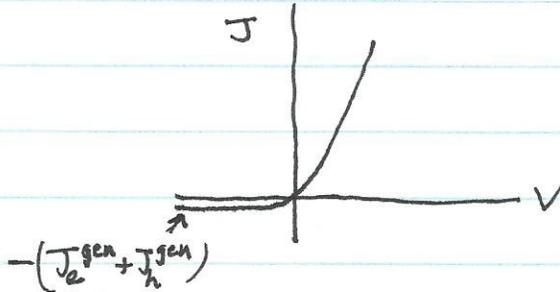
similarly, for holes

$$J_h(p \rightarrow n) = \phi_h e^{-eV_{bi}/\tau} \rightarrow \phi_h e^{-eV_{bi}/\tau} e^{eV/\tau} = J_h^{gen} e^{eV/\tau}$$

$$= J_h^{gen} e^{eV/\tau}$$

$$J_h = J_h(p \rightarrow n) - J_h(n \rightarrow p) = J_h^{gen} (e^{eV/\tau} - 1)$$

$$J = J_e + J_h = (J_e^{gen} + J_h^{gen}) (e^{eV/\tau} - 1)$$



pn jcn diode rectifier.

## Non-equilibrium Semiconductors

Non-equilibrium = excess carriers (above equilib. pop.)

Recombination:  $10^9$  to  $10^3$  s ← governs time it takes for e's to be in ~~diffusive~~ equilib with h's  
 e's reach thermal equilib in  $\sim 10^{-12}$  s  
 h's " " " " "

but e's not in diffusive equilib with each other, ∴ e's, h's have different  $\mu$ 's :

$$f_c(E, \tau) \equiv \frac{1}{1 + e^{(E - \mu_c)/kT}} \quad f_v(E, \tau) \equiv \frac{1}{1 + e^{(E - \mu_v)/kT}}$$

So

$$n_c = \frac{n_c}{\Gamma(3/2)} \int_0^{\infty} dx \frac{x^{1/2}}{e^{x - \eta_c} + 1} \equiv n_c I(\eta_c), \quad n_h = n_v I(\xi_v)$$

$$\eta_c \equiv \frac{\mu_c - E_c}{kT}$$

$$\xi_v \equiv \frac{E_v - \mu_v}{kT}$$

Joyce-Dixon approx:

$$\eta_c = \ln\left(\frac{n_c}{n_c}\right) + \frac{1}{\sqrt{8}} \frac{n_c}{n_c} - \left(\frac{3}{16} - \frac{\sqrt{3}}{8}\right) \left(\frac{n_c}{n_c}\right)^2 + \dots$$

$$\xi_v = \ln\left(\frac{n_h}{n_v}\right) + \frac{1}{\sqrt{8}} \frac{n_h}{n_v} - \left(\frac{3}{16} - \frac{\sqrt{3}}{8}\right) \left(\frac{n_h}{n_v}\right)^2 + \dots$$

Non-degenerate:

$$\eta_c \approx \ln\left(\frac{n_c}{n_c}\right) \quad (n_c/n_c \ll 1) \quad (\text{or } n_c = n_c e^{(\mu_c - E_c)/kT})$$

$$\xi_v \approx \ln\left(\frac{n_h}{n_v}\right) \quad (n_h/n_v \ll 1) \quad (\text{or } n_h = n_v e^{(E_v - \mu_v)/kT})$$

$$\vec{J}_e = \tilde{\mu}_e n_e \vec{\nabla} \mu_e \quad \vec{J}_h = \tilde{\mu}_h n_h \vec{\nabla} \mu_h$$

or, with

$$\eta_c \equiv \frac{\mu_c - \epsilon_c}{\tau} \quad \zeta_v \equiv \frac{\epsilon_v - \mu_v}{\tau}$$

$$\vec{J}_e = \tilde{\mu}_e n_e \tau \vec{\nabla} \eta_c + \tilde{\mu}_e n_e \vec{\nabla} \epsilon_c \quad \vec{J}_h = -\tilde{\mu}_h n_h \tau \vec{\nabla} \zeta_v + \tilde{\mu}_h n_h \vec{\nabla} \epsilon_v$$

$$\epsilon_c = \epsilon_{c0} - e\phi(x) \quad \epsilon_v = \epsilon_{v0} - e\phi(x) \quad \vec{E} = -\vec{\nabla} \phi$$

$$\vec{J}_e = \tilde{\mu}_e n_e \tau \vec{\nabla} \eta_c + e \tilde{\mu}_e n_e \vec{E} \quad \vec{J}_h = -\tilde{\mu}_h n_h \tau \vec{\nabla} \zeta_v + e \tilde{\mu}_h n_h \vec{E}$$

( $\sigma_e \equiv e \tilde{\mu}_e n_e$ ,  $\sigma_h \equiv e \tilde{\mu}_h n_h$ ) ← conductivities

Define  $D_e, D_h$  (diffusion coefficients) so that

$$e D_e \vec{\nabla} n_e \equiv \tilde{\mu}_e n_e \tau \vec{\nabla} \eta_c \quad e D_h \equiv \tilde{\mu}_h n_h \tau \vec{\nabla} \zeta_v$$

Then

$$\vec{J}_e = e \tilde{\mu}_e n_e \vec{E} + e D_e \vec{\nabla} n_e \quad \vec{J}_h = e \tilde{\mu}_h n_h \vec{E} - e D_h \vec{\nabla} n_h$$

In non-degenerate limit

$$\vec{\nabla} \eta_c = \frac{\vec{\nabla} n_e}{n_e} \quad \vec{\nabla} \zeta_v = \frac{\vec{\nabla} n_h}{n_h}$$

so

$$e D_e \approx \tilde{\mu}_e \tau \quad e D_h \approx \tilde{\mu}_h \tau \quad (\text{non-degenerate limit})$$