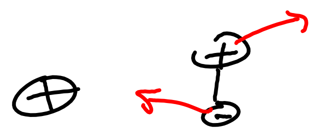
Lecture 35 Shadowitz chapter 7

Review of electric fields in matter: where have we been?

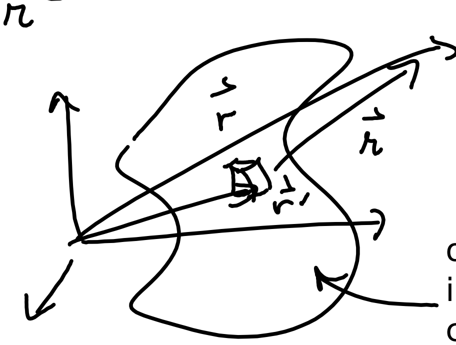
- Forces on dipoles



$$\vec{F} = (\vec{p}\cdot\vec{\nabla})\vec{E}$$

- Dipole field $V = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$

- field from a collection of dipoles

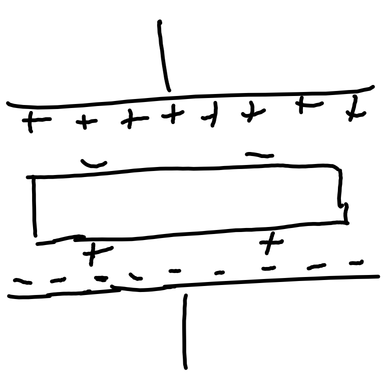


only dipoles inside; no free charge

$$V = \int \frac{1}{4\pi\epsilon_0}\frac{\rho\, d\tau'}{r} \xrightarrow{\text{no free charge}} \frac{1}{4\pi\epsilon_0}\int \frac{\vec{P}\cdot\hat{r}}{r^2}\, d\tau'$$

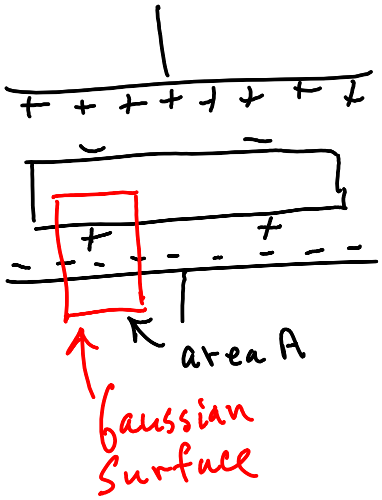
Voltage from dipoles in a small region summed

$$V_{dipoles} = \frac{1}{4\pi\epsilon_0}\left[\oint \frac{\overbrace{\vec{P}\cdot\hat{n}}^{\sigma_b}\, da'}{r} + \int \frac{\overbrace{-\vec{\nabla}\cdot\vec{P}}^{\rho_{bound}}\, d\tau'}{r}\right]$$



$$E_{tot} = \overbrace{\frac{\sigma_f}{\epsilon_0}}^{E_0} - \frac{\sigma_b}{\epsilon_0} \qquad \sigma_b = \vec{P}\cdot\hat{n} \xrightarrow[\text{material}]{\text{linear}} \epsilon_0 \chi_e E_{tot}$$

$$E_{tot} = \frac{E_0}{1+\chi_e} \quad ; \quad \vec{P} = \epsilon_0 \chi_e E_{tot}$$

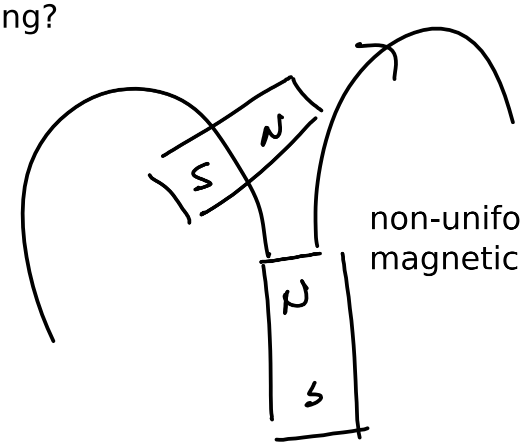


$$D = \epsilon E \qquad \oint \vec{D}\cdot d\vec{a} = Q_f$$

$$DA = \sigma_f A \qquad D = \sigma_f = \overbrace{\epsilon_0(1+\chi_e)}^{\epsilon}E$$

$$E = \frac{\sigma_f}{\epsilon_0(1+\chi_e)} = \frac{E_0}{1+\chi_e}$$

Magnetic fields in matter: where are we going?



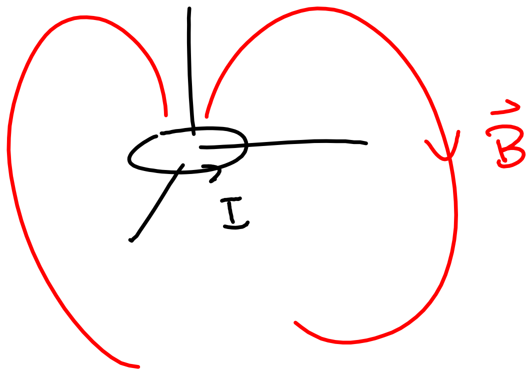
- Forces on dipoles
- magnetic dipole field B_{dipole}
- field from a collection of dipoles

Defns: $\vec{m}$ magnetic dipole moment $= I\vec{a}$

area $\vec{a} = |\vec{a}|\hat{n}$

I

$\vec{A}_{dipole}$



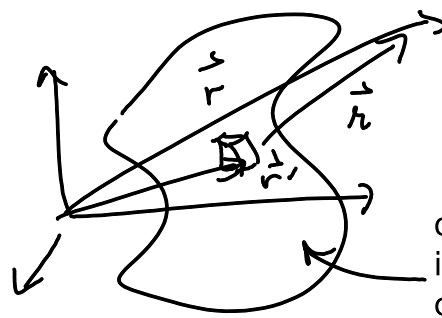
$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}')\,d\tau'}{\mathscr{r}}$$

Setting this integral up was an exam problem.

vector expression for $\vec{A}_{dipole} = \frac{\mu_0}{4\pi}\,\frac{\vec{m}\times\hat{\mathscr{r}}}{\mathscr{r}^2}$

$$\vec{M} = \frac{\text{magnetic dipole mom}}{\text{vol}}$$

$$= N\vec{m}$$



only dipoles inside; no free currents

Questions:

-causal/creative: What simple example illustrates this model?

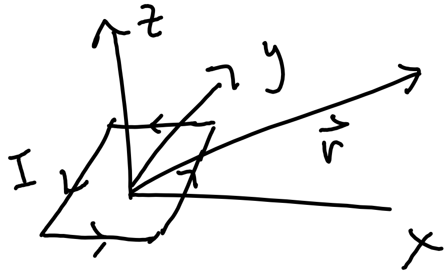
-analogous: P was used to find bound charge. What is M used to find?

-informational: What happens when different materials are put in a non-uniform B?

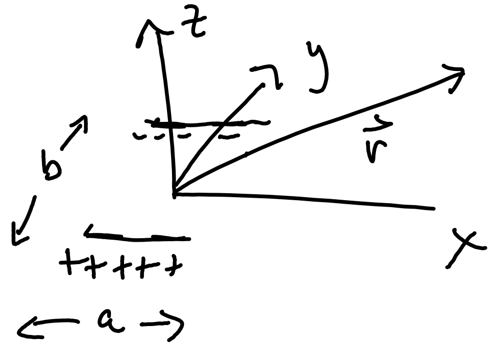
-analogy: What is analogous to an induced electric dipole in magnetic materials?

-causal/creative: How does temperature effect the magnetic dipole moment/vol.

Homework problem 1.) Solve for the dipole vector potential in the x direction from a rectanglular wire carrying current I using a direct analogy with the electric dipole field from charge along one segment of this square.



analogy $\longrightarrow$



$$A_x = \frac{\mu_0}{4\pi}\int \frac{J_x\,d\tau'}{r}$$

$$= \frac{\mu_0}{4\pi}\int \frac{dx'}{|\vec{r}-\vec{r}'|}$$

We want an approx for Ax far away.

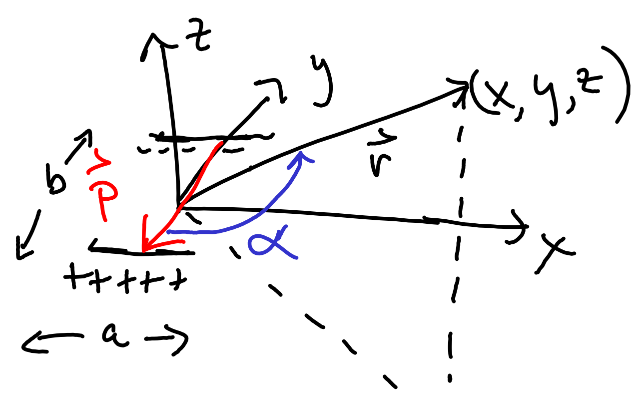
$$V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho\,d\tau'}{r} = \frac{\lambda}{4\pi\epsilon_0}\int \frac{dx'}{|\vec{r}-\vec{r}'|}$$

We want an approx for V far away.

$$p = qb = \lambda ab$$

$$V_{dipole} = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$$

$$\vec{p}\cdot\hat{r} = p\cos\alpha = p\left(-\frac{y}{r}\right)$$



Fill in the analogous relations to get

$$A_x = -\frac{m}{4\pi c^2}\frac{y}{r^3}$$

$$c^2 = \frac{1}{\mu_0\epsilon_0}$$

$m = Iab$ is the magnetic dipole moment.

Also

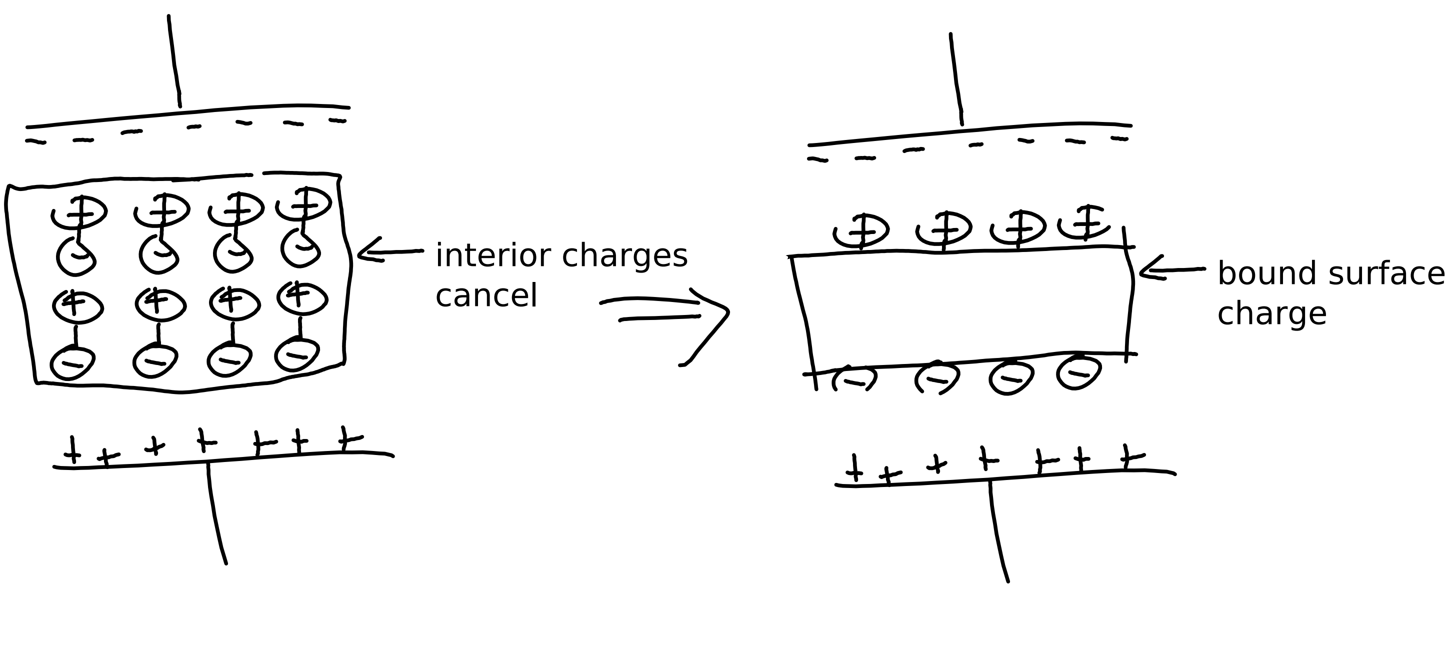
$$A_y = \frac{Iab}{4\pi\epsilon_0 c^2}\frac{x}{r^3};$$

$$A_z = 0$$

Homework problem 2: Derive an expression for the magnetic field from this dipole vector potential.

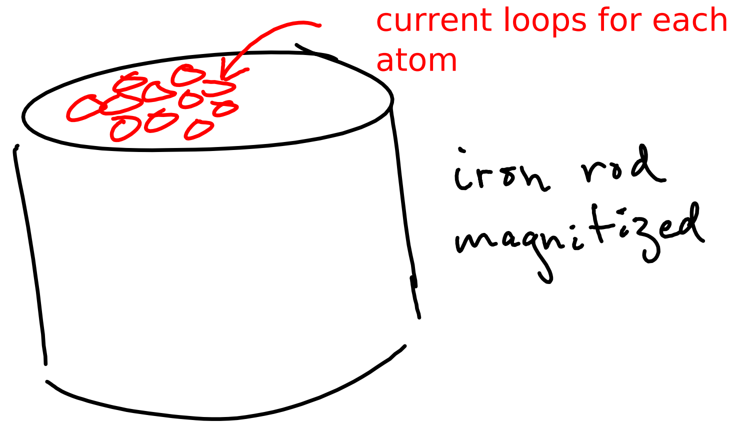
-causal/creative: What simple example illustrates this model?

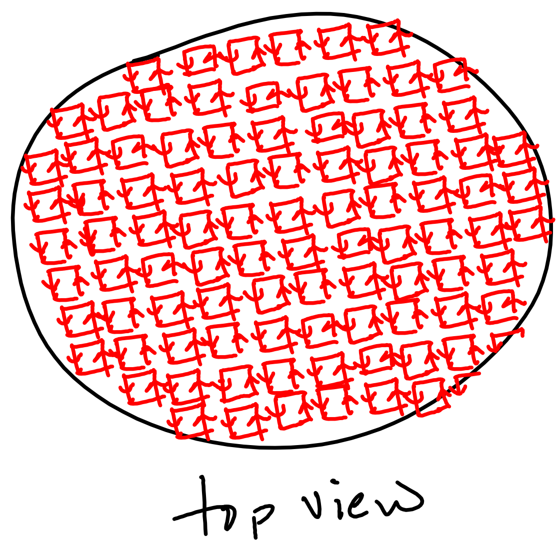
Simple model to illustrate electric fields in matter



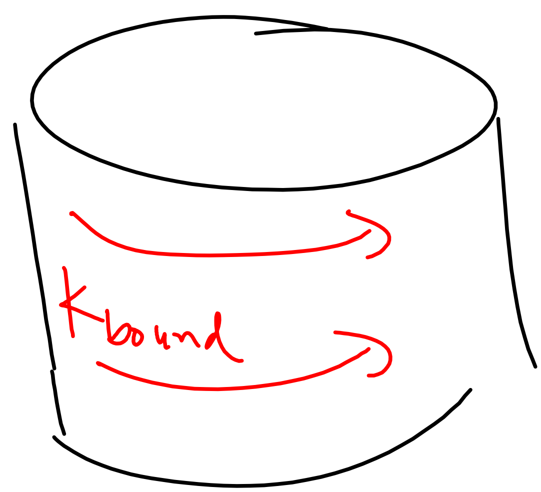
Simple model to illustrate magnetic fields in matter:

Start with magnetic dipoles





interior currents
cancel leaving only
surface currents



Throw away the iron and use just the bound currents to calculate B everywhere.

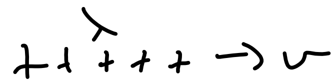
informational: What field is generated inside and outside this magnitized iron rod?

ans: same as a solenoid.

For the big big D look up youtube hms pinafore I am the captain

Review of some defns of

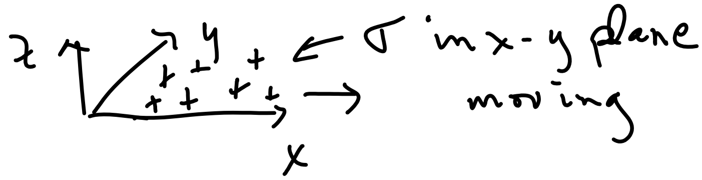
charges moving along a line



$$dq = \lambda\, dx$$

$$I = \frac{dq}{dt} = \lambda \frac{dx}{dt} = \lambda v$$

charges moving in a plane

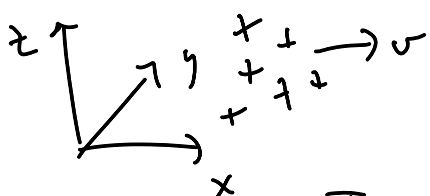


$$dq = \sigma\, dx\, dy$$

$$I = \frac{dq}{dt} = \sigma \frac{dx}{dt} dy = K\, dy \qquad \vec{K} = \sigma \vec{v}$$

surface current density

charges moving in a volume



$$dq = \rho\, dx\, dy\, dz$$

$$I = \frac{dq}{dt} = \rho \frac{dx}{dt} dy\, dz = \rho v\, dx\, dy = J\, da$$

$$\vec{J} = \rho \vec{v}$$

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{dt} \qquad \oint \vec{J} \cdot da = -\frac{dQ_{enclosed}}{dt}$$

congruous: how do we calculate A for a collection of dipoles?

analogy: What does the analogous result look like for electric dipoles?

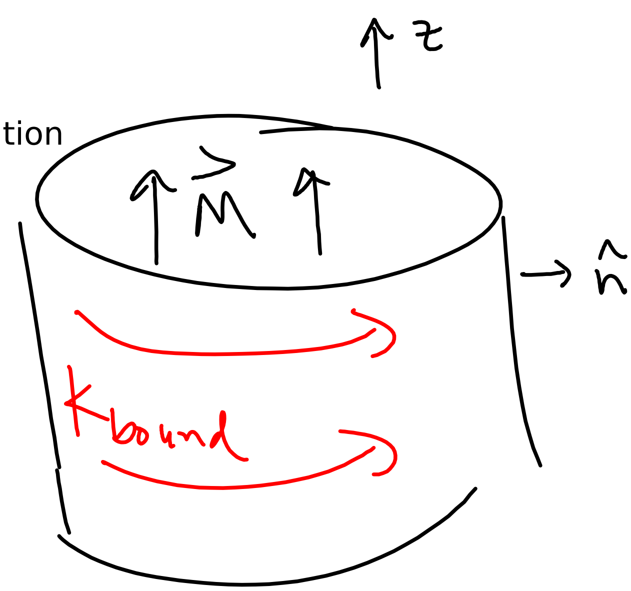
$$V = \frac{1}{4\pi\epsilon_0}\left[\oint \frac{\overbrace{\vec{P}\cdot\hat{n}}^{\sigma_b}\,da'}{r} + \int \frac{\overbrace{-\vec{\nabla}\cdot\vec{P}}^{\rho_{bound}}\,d\tau'}{r}\right]$$

After a some vector calculus it can be shown that the sum over all dipole fields in a chunk of matter which has magnetic dipoles distributed throughout is

$$\vec{A}_{dipoles} = \frac{\mu_0}{4\pi}\left[\int \frac{\vec{J}_b\,d\tau}{r} + \oint \frac{\vec{K}_b\,da}{r}\right]$$

$$\vec{J}_b = \vec{\nabla}\times\vec{M} \qquad \vec{K}_b = \vec{M}\times\hat{n}$$

Here is the interpretation for a magnetized iron rod.



Homework problem 3.) If the rod above is very long and has $\vec{M} = \beta r\,\hat{z}$

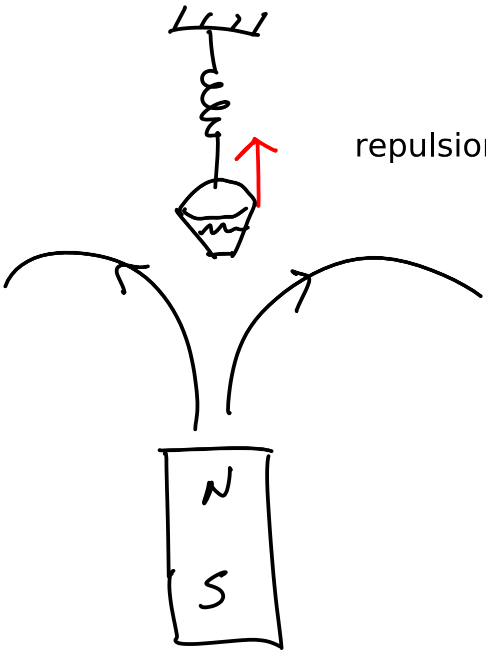
(a) find both the surface and volume current densities.
(b) find the magnetic field.

-informational: What happens when different materials are put in a non-uniform B?

-causal/creative; What simplification can be made below to understand the phenomina?

-analogy: What is analogous to an induced electric dipole in magnetic materials?

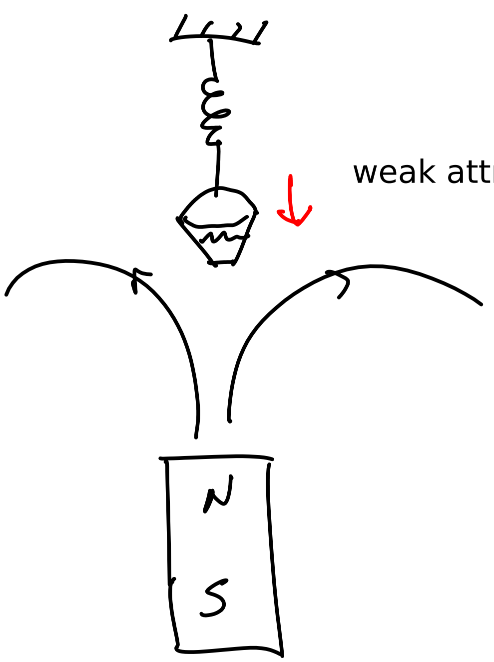
diamagnetism



repulsion

creative/causal: How do I simplify this to understand the repulsion?

paramagnetism



weak attraction

modifying: What parameters could I vary to determine the cause?

see youtube video on levitating a frog.

Ferromagetism