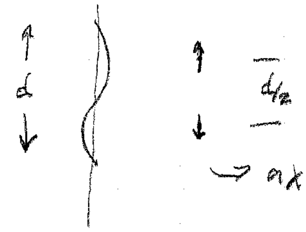
Antenna arrays.

- can approximate continuous current on antenna by dipoles:



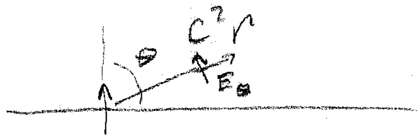

→ axial quadrupole.

- rotate dipole direction ↑ ↓ non axial.
- arrays: equal spacing
  → relevant to models of diffraction.

Sum up fields from individual dipoles.

Axial case:

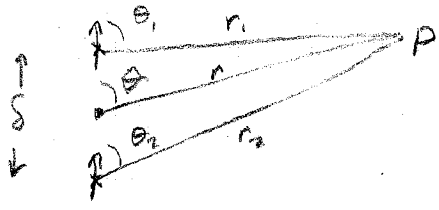
$$E_\theta = \frac{[\ddot{p}]}{c^2 r}\sin\theta \rightarrow -\frac{\omega^2 p_0}{c^2 r}\sin\theta\, e^{-i\omega t'}$$

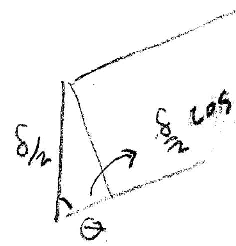



$t'$ = retarded time.

2 dipoles: separation $\delta$

relative phase $\Delta\phi$




$$E_\theta = -\frac{\omega^2 p_0}{c^2}\left(\frac{\sin\theta_1}{r_1}e^{-i\omega t_1'} + \frac{\sin\theta_2}{r_2}e^{-i(\omega t_2' - \Delta\phi)}\right)$$

$t_1' = r_1/c \quad t_2' = r_2/c$

$r_1 = \left(r^2 + (\delta/2)^2 - 2r(\delta/2)\cos\theta\right)^{1/2}$ (law of cosines)

$r_2 = \left[r^2 + (\delta/2)^2 - 2r(\delta/2)\cos(\pi-\theta)\right]^{1/2}$

$r_{1/2} = \left[r^2 + (\delta/2)^2 \mp r\delta\cos\theta\right]^{1/2}$

Now make approximations: $\delta << r$ , $\theta_1 \approx \theta_2 \approx \theta$

$$E_\theta = -\frac{\omega^2 p_0}{c^2 r} \sin\theta \left[ e^{-ikr\left(1 + \frac{1}{8}\frac{\delta^2}{r^2} - \frac{\delta}{2r}\cos\theta\right)} + e^{-i\left[kr\left[1+\frac{1}{8}\frac{\delta^2}{r^2} - \frac{\delta}{2r}\cos\theta\right] - \Delta\right.} \right.$$

$$= -\frac{\omega^2 p_0}{c^2 r} \sin\theta \, e^{-ik\left(r + \frac{\delta^2}{8r}\right)} \left[ e^{ik\frac{\delta}{2}\cos\theta} + e^{-i\left(k\frac{\delta}{2}\cos\theta - \Delta\phi\right)} \right]$$

$$= -\frac{2\omega^2 p_0}{c^2 r} \sin\theta \, e^{ik\left(r + \frac{\delta^2}{8r}\right)} e^{i\Delta\phi/2} \cos\left(k\frac{\delta}{2}\cos\theta - \frac{\Delta\phi}{2}\right)$$

if we put dipoles out of phase $\Delta\phi = \pi$ note $\frac{e^{ikr}}{r}$ spherical wave.

$[\quad] \rightarrow 2i\sin\left(k\frac{\delta}{2}\cos\theta\right)$ as in book.

pattern is symmetric around $\theta = 0$ axis

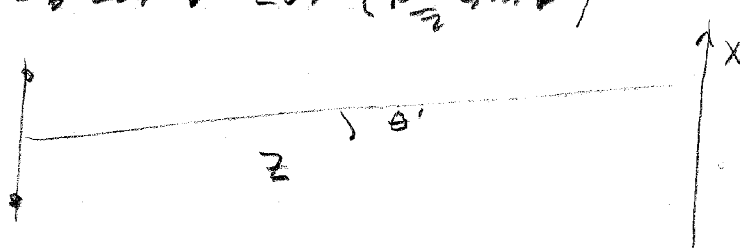
$B_\phi = E_\theta$ (radiation zone), so

$$I = \langle \vec{S}_{rad} \rangle \cdot \hat{n} = \frac{c}{8\pi} Re\left(E_\theta B_\phi^*\right) = \frac{4\omega^4 p_0^2}{c^3 r^2}\frac{1}{8\pi} \sin^2\theta \cos^2\left(\frac{k\delta}{2}\cos\theta - \frac{\Delta\phi}{2}\right)$$

dipole pattern $\cdot (2)^2 \cdot$ angular modulation

For better comparison to interference / diffraction, write in terms of $\theta' = \frac{\pi}{2} - \theta$ , let $\Delta\phi = 0$

$$I = I_0 \cos^2\theta' \cos^2\left(k\frac{\delta}{2}\sin\theta'\right)$$




$\sin\theta' = x/z$

$$\therefore I \propto \cos^2\theta' \cos^2\left(\frac{k\delta}{2z}x\right)$$

interference fringes as in double slit.

changing $\Delta\phi$ moves the fringe pattern along $x$

$\Delta\phi = 0$ gives peak at $x=0$

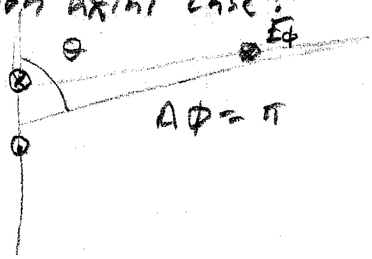
$\Delta\phi = \pi$ " zero at $x=0$ e.g.

can compare $\Delta\phi = \pi$ case to full wave and quadrupole $(\delta \to 0)$

- scattering will tend to $\Delta\phi = 0$

Polarization effects:

- non axial case:




observe in plane.

$$E_\phi = -\frac{\omega^2 p_0}{c^2 r} e^{-i\omega t'}$$

no sin factor.

here it's possible to get radiation along $\theta = 0$ line

- "end-fire"

Antenna array

same far-field approximation:

$$E_\theta = -\frac{\omega^2 p_0}{c r} \underbrace{\sin\theta}_{\text{depends on polarization.}} \sum_j e^{-i\omega t_j} e^{i\Delta\phi_j}$$

special cases: $\Delta\phi_j = 0$ (sources are all in phase)

regular spacing: $\delta$

far field $\frac{k\cdot(N\delta)^2}{r} \ll 2\pi$

$$\rightarrow E_\theta = -\frac{\omega^2 p_0}{c} \sin\theta \, \frac{e^{-ikr}}{r} \sum_{j=-N/2}^{N/2} e^{ik\delta(\frac{1}{2}+j)\cos\theta}$$

same as diffraction grating.