

11-28-07

Note Title

11/13/2006

## Chladni figures demo

Degeneracy

Eigenfrequencies  
Normal Mode  
frequencies

we showed last time  
that

$$\omega_{n,m}^2 = c^2 \pi^2 \left( \frac{n^2}{L_x^2} + \frac{m^2}{L_y^2} \right)$$

So if the drum is  
square, then  $L_x = L_y \equiv L$   
and

$$\omega_{n,m}^2 = \left( \frac{c\pi}{L} \right)^2 (n^2 + m^2)$$

this says that

$$\omega_{N,m}^2 = \omega_{m,N}^2$$

i.e. 2 modes have the same frequency:

$N,m$  mode =

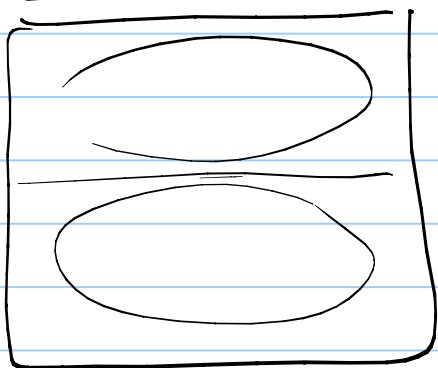
$$\sin\left(\frac{N\pi x}{L}\right) \sin\left(\frac{m\pi y}{L}\right)$$

$m,N$  mode =

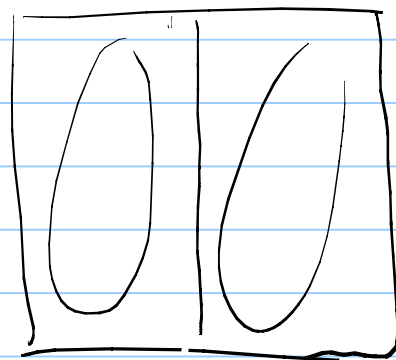
$$\sin\left(\frac{m\pi x}{L}\right) \sin\left(\frac{N\pi y}{L}\right)$$

E.g.

1, 2



2, 1



what happens when 2 modes compete?

Rather than continue on to  $\nabla^2$  in 3 cartesian coordinates, lets jump to spherical coordinates.

See wiki link to mathworld for derivation

## Laplacian in spherical coordinates.

$$\nabla^2 \psi(x, y, z) = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}$$

$$\nabla^2 \psi(r, \theta, \phi) = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \psi}{\partial r} \right)$$

$$+ \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \theta} \left( \sin^2 \theta \frac{\partial \psi}{\partial \theta} \right)$$

$$+ \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \psi}{\partial \phi^2}$$

$$\nabla^2 \psi = 0 \quad \text{Laplace Equation}$$

As usual we suppose

$$\psi(r, \theta, \phi) = R(r)P(\theta), Q(\phi) \Rightarrow$$

$$\nabla^2 \psi = \frac{1}{r^2} P Q \frac{d}{dr} \left( r^2 \frac{dR}{dr} \right)$$

$$+ \frac{R Q}{r^2 \sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{dP}{d\theta} \right)$$

$$\frac{R P}{r^2 \sin^2 \theta} \frac{d^2 Q}{d\phi^2} = 0$$

Divide by  $R P Q = \psi$

$$\frac{1}{r^2 R} \frac{d}{dr} (r^2 R') + \frac{1}{P r^2 \sin \theta} \frac{d}{d\theta} (\sin \theta P')$$

$$+ \frac{1}{Q r^2 \sin^2 \theta} Q'' = 0$$

multiply by  $r^2 \sin^2 \theta$

$$\frac{\sin^2 \theta}{R} \frac{d}{dr} (r^2 R') + \frac{\sin \theta}{\rho} \frac{d}{d\theta} (\sin \theta P')$$
$$= -Q''$$

