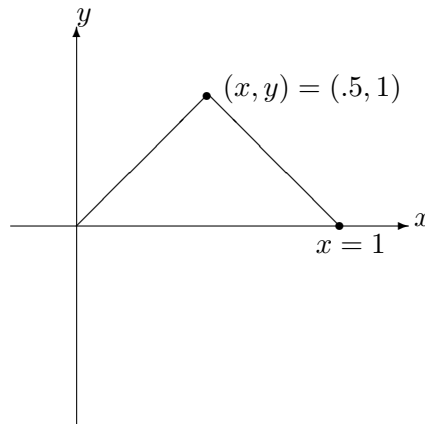


In order to receive full credit, **SHOW ALL YOUR WORK**. Full credit will be given only if all reasoning and work is provided. When applicable, please enclose your final answers in boxes.

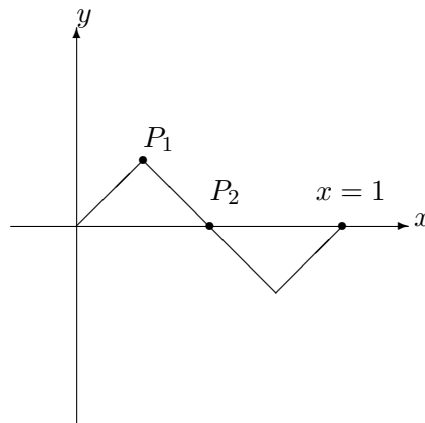
1. (10 Points) Conceptual Questions. For the following questions assume that we are considering the physical problem on a bounded domain,  $x \in [0, 1]$ .

(a) Write down the heat and wave equations and also any initial conditions needed for unique solutions.

(b) Assume that the following graph is the initial temperature for a homogeneous heat problem with boundary conditions,  $u_x(0, t) = 0$ ,  $u_x(1, t) = 0$ . Describe the physical meaning of these boundary conditions and graph the temperature profile for  $t \rightarrow \infty$ .



(c) Assume that the following graph is the initial displacement of an elastic string modeled by the homogeneous wave equation subject to boundary conditions  $u(0, t) = 0$ ,  $u(1, t) = 0$ . Describe the physical meaning of these boundary conditions and describe time-dynamics of the points,  $P_1$  and  $P_2$ , on this elastic string assuming that the string has no initial velocity.



2. (10 Points) Quick Problems

- (a) Apply separation of variables to  $u_t = u_{xx} + u_{yy}$  and find three ordinary differential equations consistent with the PDE.

- (b) Given,

$$F''(x) + \lambda F(x) = 0, \quad \lambda \in [0, \infty).$$

The following table contains different boundary conditions for the ODE. Fill in each table element with either a yes or a no.

	Boundary value problem has a cosine solution	Boundary value problem has a sine solution	Boundary value problem has a nontrivial constant solution
$F'(0) = 0, F'(L) = 0$			
$F(0) = 0, F'(L) = 0$			
$F(0) = 0, F(L) = 0$			
$F'(0) = 0, F(L) = 0$			

3. (10 Points) Show that  $u(x, y, z) = (x^2 + y^2 + z^2)^{-1/2}$  is a solution to  $u_{xx} + u_{yy} + u_{zz} = 0$ .

4. (10 Points) Given

$$F'' + \lambda F = 0, \quad \lambda \in [0, \infty). \tag{1}$$

Find all nontrivial solutions  $F$  and constants  $\lambda$  that satisfy the boundary conditions:

(a)  $F'(0) = 0$  and  $F'(L) = 0$

(b)  $F'(0) = 0$  and  $F(L) = 0$

5. (10 Points) Solve the following PDE.

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad (2)$$

$$u(0, t) = 0, \quad u(\pi, t) = 0, \quad (3)$$

$$u(x, 0) = f(x), \quad (4)$$

$$u_t(x, 0) = g(x) \quad (5)$$