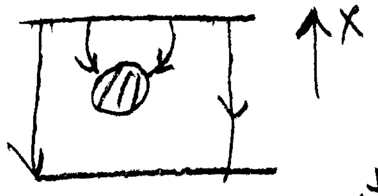
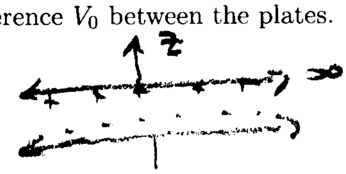


toward upper plate

4. A conducting sphere is fixed between the plates of a parallel plate capacitor. Batteries fix the potential between the lower plate ($V = 0$) and upper plate at V_0 and the sphere at V_s. The sphere is then moved a small distance Δx. Using field energy concepts, describe how you would calculate the work required to move the sphere this distance?

Solve Laplace's eqn for $V(x,y,z)$. Find $\vec{E} = -\vec{\nabla} V$
$= -(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z})V$. Calculate field energy $= \frac{1}{2}\epsilon_0 \int E^2 d\tau$
then repeat for displaced sphere.
Difference in field energy is work required to move sphere
For this geometry the relaxation method is simple.

5. An infinite vacuum parallel plate capacitor has a voltage difference V_0 between the plates. Derive the separation of variables solution to Laplace's equation.



$V(x,y,z) = X(x)\,Y(y)\,Z(z)$ $\quad \nabla^2 V = 0$

$\underbrace{YZ\frac{d^2X}{dx^2}}_{c_1} + \underbrace{XZ\frac{d^2Y}{dy^2}}_{c_2} + \underbrace{XY\frac{d^2Z}{dz^2}}_{c_3} = 0$

$Y(y)$ can't depend on y since changing y yields same result $\Rightarrow C_2 = 0$
Same reason yields $C_1 = 0$

$XY\frac{d^2Z}{dz^2} = 0$ or O.D.E. $\frac{d^2Z}{dz^2} = 0$ $\quad Z(z) = mz + b$ at $z=0$

$m = \frac{V}{d}$, $b = 0$

6. (Extra Credit) How would you construct a wave function for the electron in a hydrogen atom to "orbit" the proton as the Earth orbits the Sun?

Solve Sch eqn for hydrogen atom using separation of variables. Sum the soln's to give a wavepacket orbiting the proton at $t=0$. This gives the motion or orbit of the packet for all time.