HW - 8

Note Title 11/17/2007

1) Basically this has to do with the default parameters for mathematica's Fourier Series. So

in: FourierTrigSeries[sin(πx), 2]

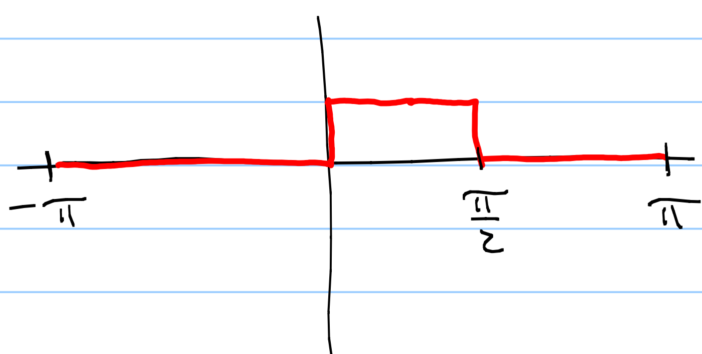
out: $\frac{8\sin(2\pi x)}{3\pi} - \frac{16\sin(4\pi x)}{25}$

But

in: FourierTrigSeries[sin(4πx), 4] ~ or any number here

out: $\underbrace{\sin(4\pi x)}$ periodic on [-.5, .5) which corresponds to mathematica's default params.

2) Boas 7: 5-2



periodic on $[-\pi, \pi]$

$l = \pi$

$$a_0 = \frac{1}{\pi}\int_0^{\pi/2} dx = \frac{1}{2}$$

$$\frac{a_0}{2} = 1/4$$

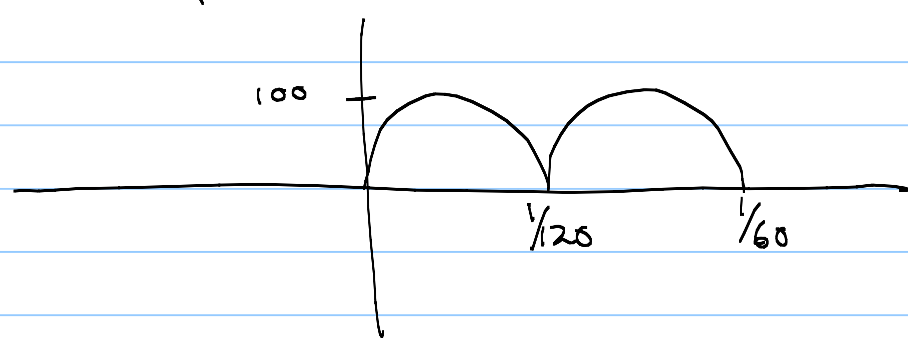
$$a_1 = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \cos(x)\,dx = \frac{\sin(x)}{\pi}\Big|_0^{\pi/2} = 1/\pi$$

$$b_1 = \frac{1}{\pi}\int_0^{\pi/2} \sin(x)\,dx = -\frac{\cos x}{\pi}\Big|_0^{\pi/2} = \frac{1}{\pi}$$

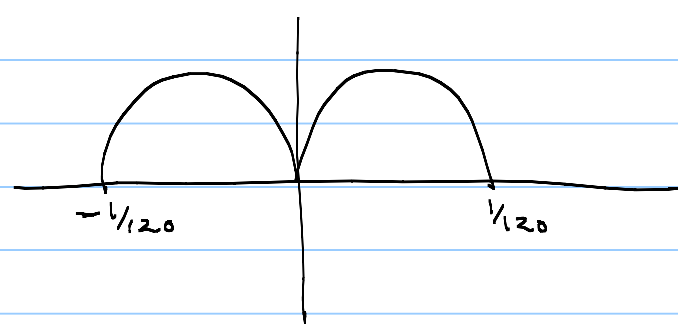
$$f(x) \approx \frac{1}{4} + \frac{\cos x}{\pi} + \frac{\sin x}{\pi} \quad ---$$

f[x_] = Piecewise[{{0, -π < x < 0}, {1, 0 < x < π/2 }, {0, ...}}]
FourierTrigSeries[f[x], x, 5, FourierParameters → {1, 1/2π}

3) 7:10-4



Same as



$l = 1/120$

So

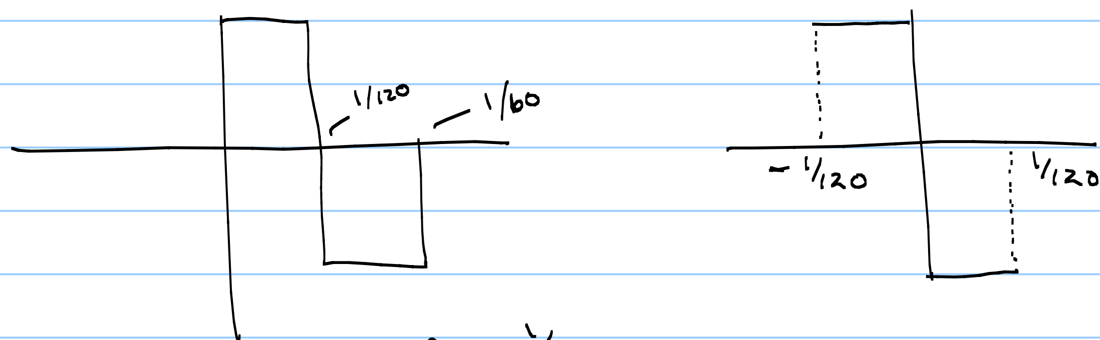
g[t] = FourierTrigSeries[f[t], t, 10, Fourier parameters → {1, 60}]

$$= 60\left(\frac{1}{30\pi} - \frac{\cos(240\pi t)}{45\pi} - \frac{\cos(480\pi t)}{225\pi} \cdots\right)$$

note $240\pi = \underbrace{\underbrace{120}_{f} \cdot 2\pi}_{\omega}$

frequencies : 0, 120 Hz, 240 Hz, . . .

4) 7: 10 - 9



$\ell = 1/120$

f[t_] = Piecewise[{{1, -ℓ < t < 0}, {-1, 0 < t < ℓ}}]
FourierTrigSeries (f[t], t, 4, FP -> {1, 1/2ℓ}]

$$= -60\left(\frac{\sin(120\pi t)}{15\pi} + \frac{\sin(360\pi t)}{45\pi} \cdots\right)$$

frequencies : 60 Hz, 180 Hz, - - -

$$f(x) = \sum_{N=-\infty}^{\infty} c_N \, e^{iN\pi x/\ell}$$

$$|f(x)|^2 = \sum_{N=-\infty}^{\infty} c_N \, e^{iN\pi x/\ell} \sum_{N'=-\infty}^{\infty} \overline{c_{N'}} \, e^{-iN' x/\ell}$$

$$= \sum_N \sum_{N'} c_N \overline{c_{N'}} \, e^{i(N-N')\pi x/\ell}$$

integrate this from $-\ell, \ell$

$$\sum_N \sum_{N'} c_N \overline{c_{N'}} \underbrace{\int_{-\ell}^{\ell} e^{i(N-N')\pi x/\ell} \, dx}_{2\ell \, \delta_{NN'}}$$

$$= \sum_N |c_N|^2 \, 2\ell$$

average over a period $= \frac{1}{2\ell} \int_{-\ell}^{\ell} (\quad) \, dx$

So average $|f(x)|^2 = \sum_{N=-\infty}^{\infty} |c_N|^2$ QED.

For 6) see class notes