MATH348 - April 6, 2009 **NAME**:

Exam II - 50 Points - 50 minutes **SECTION**:

In order to receive full credit, SHOW ALL YOUR WORK. Full credit will be given only if all reasoning and work is provided. When applicable, please enclose your final answers in boxes.

1. (10 Points) Briefly respond to the following:

   (a) Explain why a function defined on a finite portion of the real line has a Fourier series representation. Is this representation unique?

   (b) Compare and contrast Fourier series with Fourier integral. Specifically:

      i. What is the purpose of each.

      ii. How does one get from Fourier series to Fourier integral.

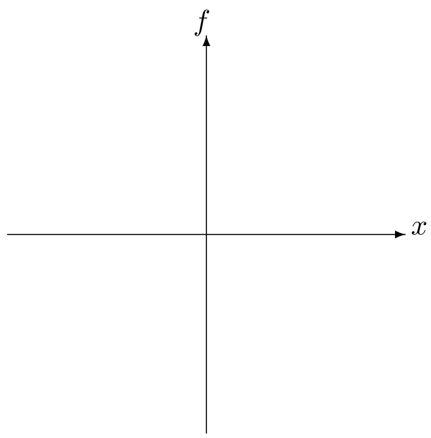
      iii. What are similarities and differences between the linear combinations of their oscillatory modes?

2. (10 Points) Let,

$$f(x) = \begin{cases} 0, & -\pi < x < 0, \\ \pi - x, & 0 \leq x < \pi. \end{cases} \tag{1}$$

Find the Fourier series representation of f.

3. (10 Points) Find the **complex** Fourier series representation of the function given on its principle period in the graph below.



4. (10 Points) Let $f(x) = \pi$ for $x \in [0, L]$. Find **both** the Fourier cosine **and** sine half-range expansions of f.

5. (10 Points)

(a) Let $\hat{f}(\omega) = \sum_{n=-\infty}^{\infty} c_n \delta(\omega - n)$. Find the inverse Fourier transform of $\hat{f}$.

(b) Suppose that $c_n = i\frac{(-1)^n}{n}$ for $n \neq 0$ and $c_0 = 0$. Graph the function $f(x) = \mathfrak{F}^{-1}\left\{\hat{f}\right\}$ below.