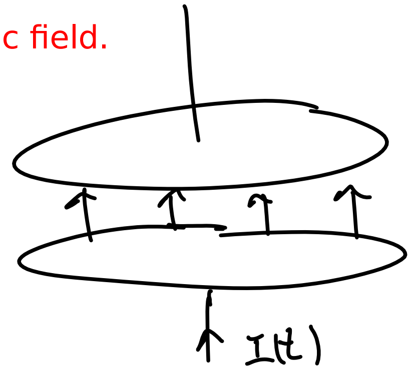
Lecture 26

Continue with the perturbation solution for oscillating current in a capacitor.

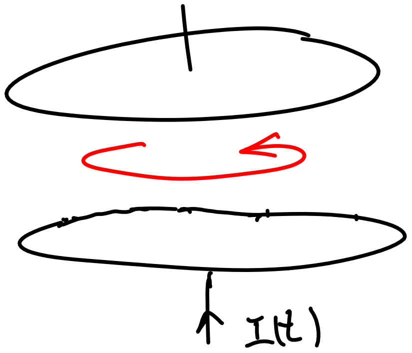
First approximation is a uniform electric field.



$$E(t) = E_1 e^{i\omega t}$$

This generates a magnetic field by Ampere's law.

First approximation is a magnetic field that varies with radius.

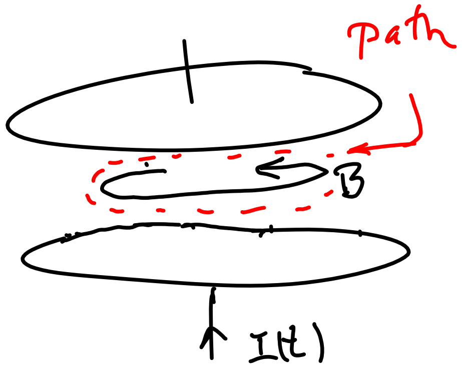


Where does this B point?

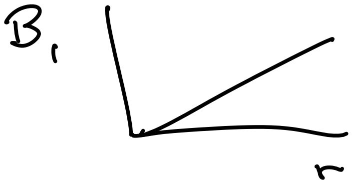
What path do we use to apply Ampere's law?

$$\oint \vec{B}_1 \cdot d\vec{r} = \mu_0 \int \vec{J}_d \cdot d\vec{a}$$

$$\begin{array}{ccc} \oint \vec{B}_1 \cdot d\vec{r} & = & \mu_0 \int \vec{J}_d \cdot d\vec{a} \\ \| & & \| \\ B_1 2\pi r & = & \epsilon_0 \dfrac{\partial (E_1 e^{i\omega t})}{\partial t} \end{array}$$



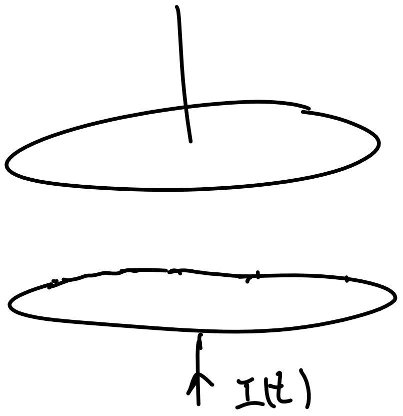
$$B_1 = \frac{i\omega}{2}\frac{r}{c^2}E_1 e^{i\omega t}$$



Second approximation for the electric field is caused by the first approximation for the changing magnetic field via Faraday's law.

Where does this E point?

What path do we use to use Faraday's law?

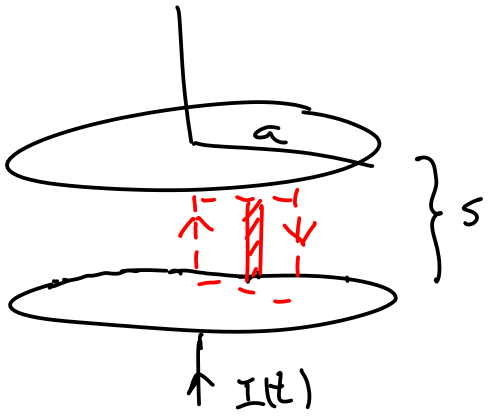


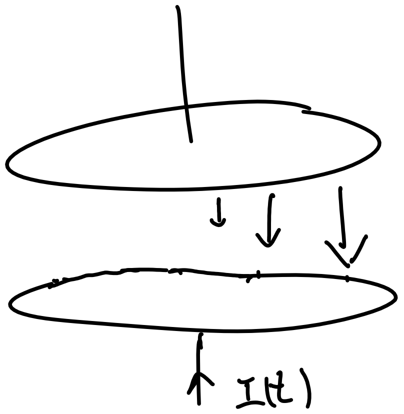
$$\oint \vec{E}_2 \cdot d\vec{r} = -\frac{\partial}{\partial t}\int \vec{B}_1 \cdot d\vec{a}$$

E_2 is zero on axis (no flux)

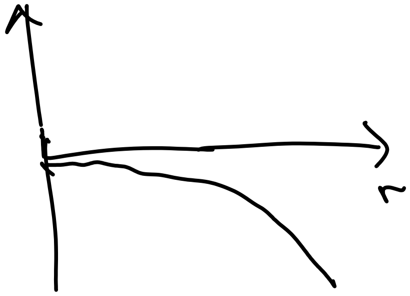
$$-E_2 s = -\frac{\partial}{\partial t}\int \frac{i\omega}{2}\frac{r E_1}{c^2} e^{i\omega t} s\,dr$$

$$E_2 = -\frac{\omega^2 r^2}{4}\frac{1}{c^2}E_0 e^{i\omega t}$$





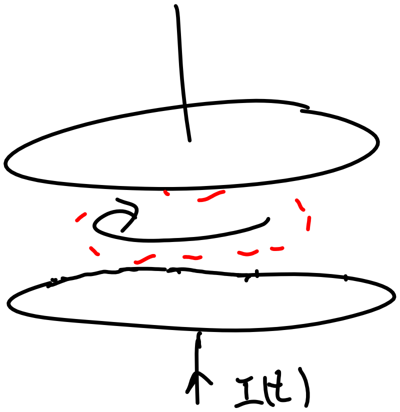
$E_2 \propto -r^2$



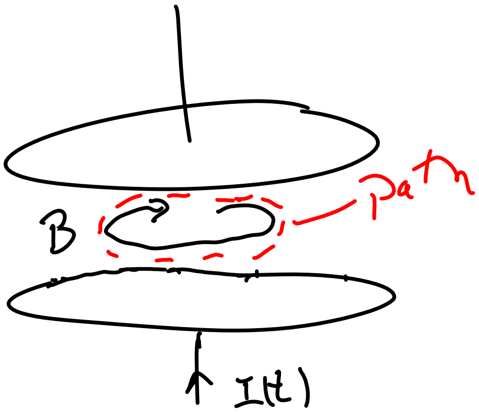
Second approximation for the magnetic field is caused by this changing electric field via Ampere's law.

Where does this B point?

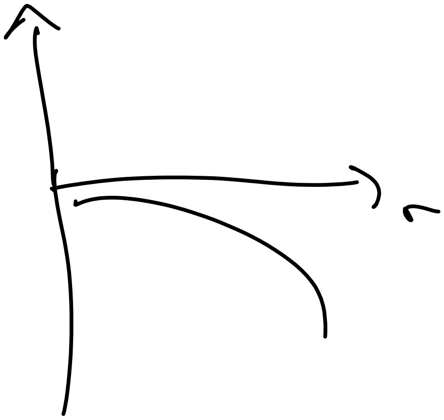
What path do we use to apply Ampere's law?



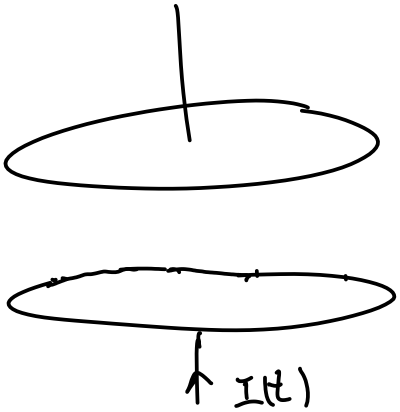
$$\oint \vec{B}_2 \cdot d\vec{r} = \mu_0 \int \epsilon_0 \frac{\partial E_z}{\partial t} \, 2\pi r \, dr$$



$B_2 \propto -r^3$



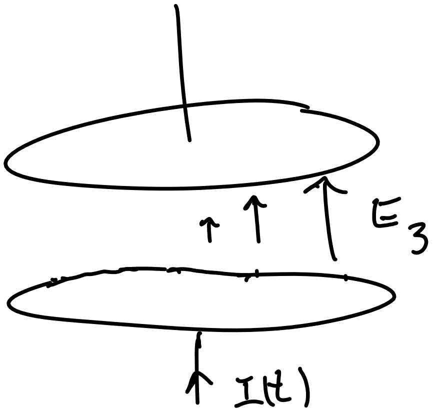
Third approximation for the electric field is due to this changing magnetic field.



Where does this E point?

What path do we use to use Faraday's law?

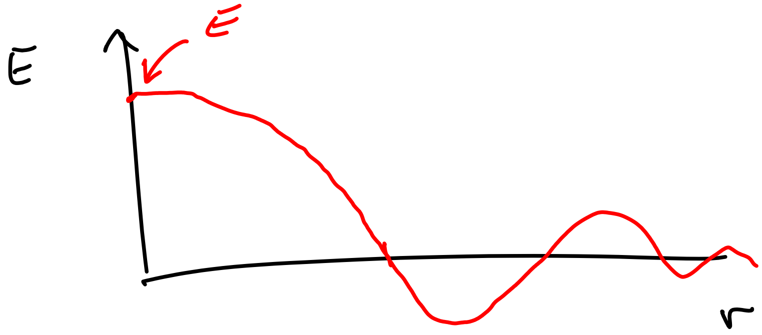
$$\oint \vec{E}_3 \cdot d\vec{r} = -\frac{\partial}{\partial t}\int \vec{B}_2 \cdot d\vec{a}$$



For the homework you are to find expressions for E_1, E_2, E_3 & B_1, B_2.

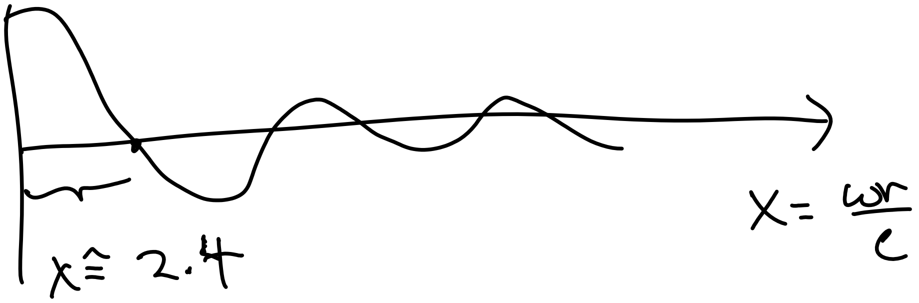
Questions:

informational: What does the total electric field look like?

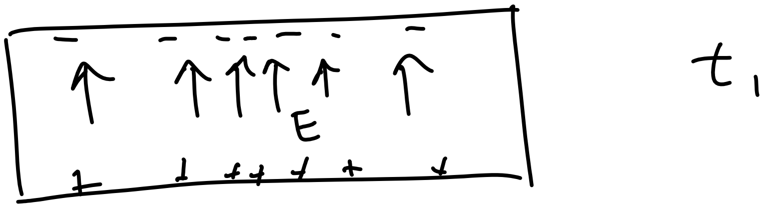


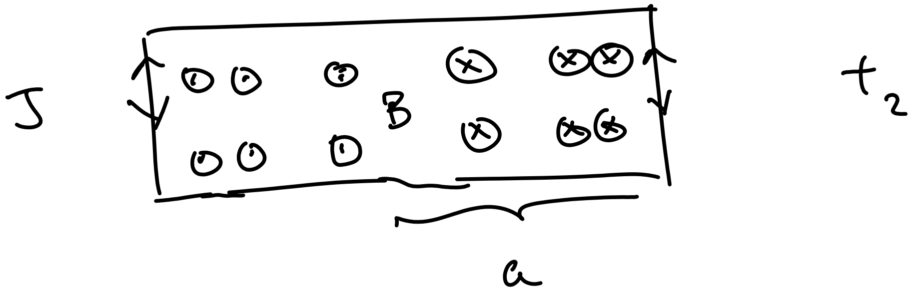
$$E_{tot} = E_1 e^{i\omega t}\left[1 - \frac{1}{2^2}\left(\frac{\omega r}{c}\right)^2 + \frac{1}{2^2 4^2}\left(\frac{\omega r}{c}\right)^4 + \cdots\right]$$

↑ dimensionless
small if $\omega r < c$



What if we close the cap with a side where E=0?



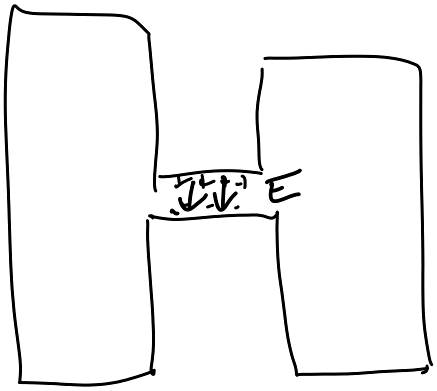


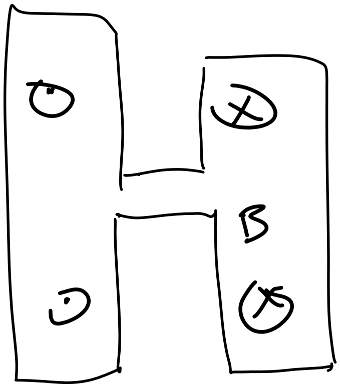
$$2.4 = \frac{\omega a}{c}$$

$$\omega = \frac{2.4(3\times 10^8)}{a}$$

Questions: informational: Can you isolate the electric and magnetic fields?

A can like this might have most E in one region and most B in another.

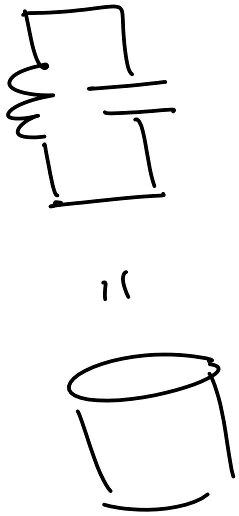




analogy: how is this similar and different to a LC oscillator?

Lumped circuit

You can use wire cutters to remove
the inductor or capacitor.

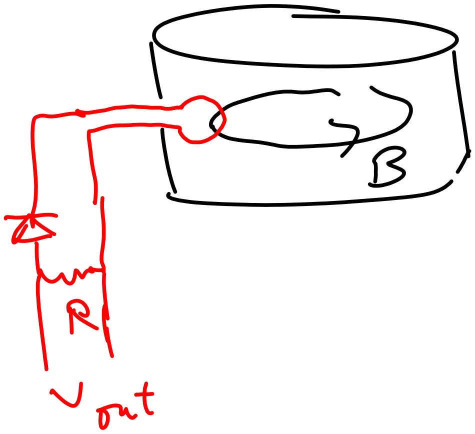


distributed circuit

You can not isolate the inductance
from the capacitance.

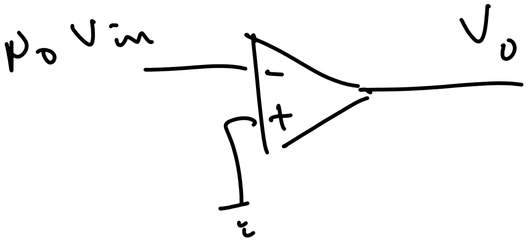
Questions:

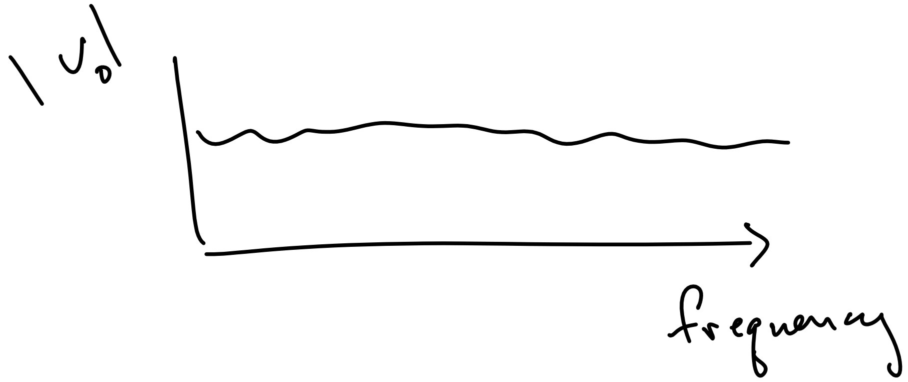
-informational: how do measure this resonant effect?

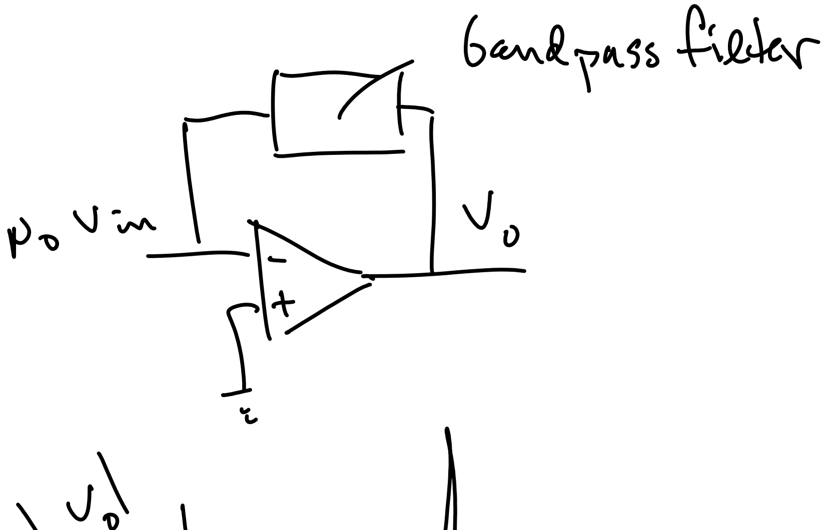


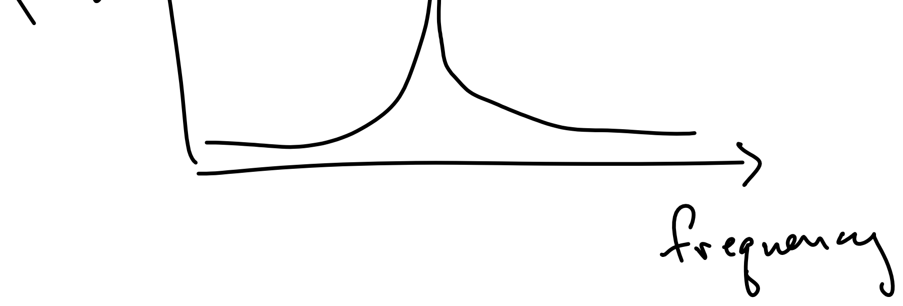
Have the wire loop aligned to maximize
the magnetic flux which is changing with
time.

-informational: how do I make signal generator that works at these frequencies?

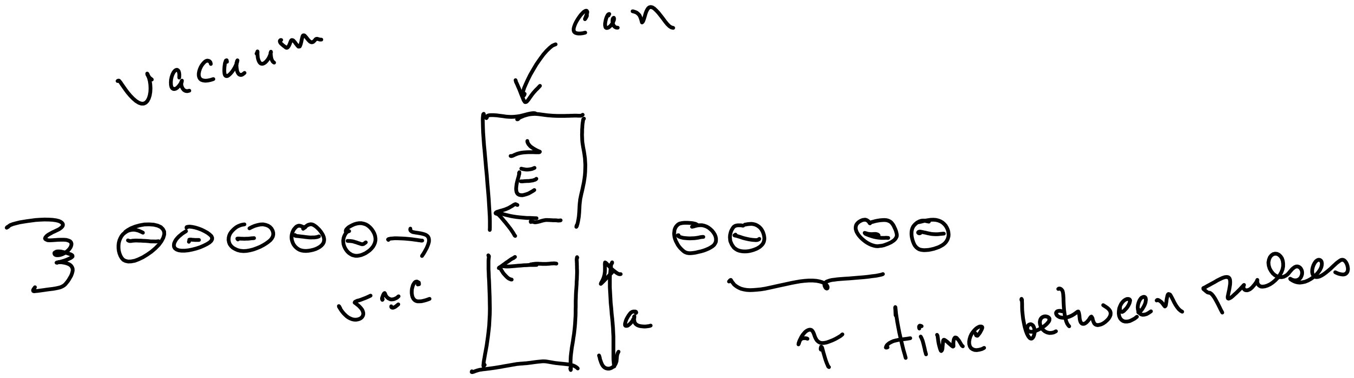






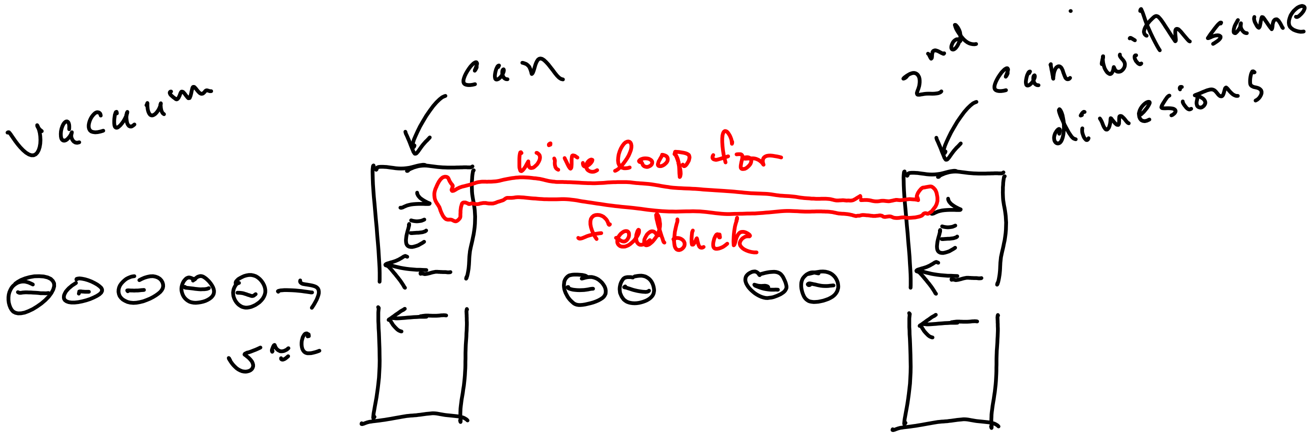


Amplifier plus feedback = oscillator

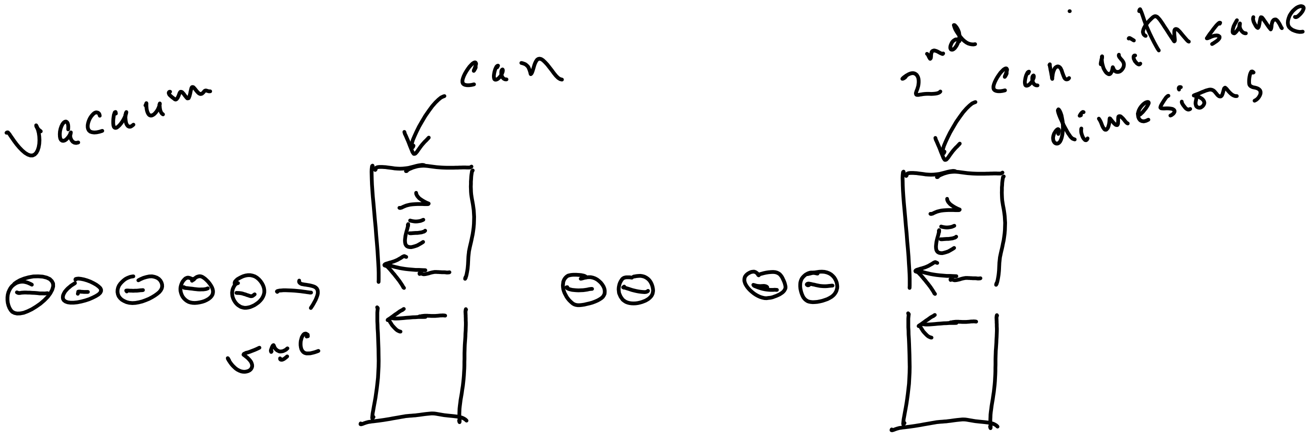


$$\nu_{\text{resonant}} = \frac{1}{\tau} = \frac{2.4(3\times10^{8})}{a}$$

Klystron



Questions:


Vacuum
can
2nd can with same dimesions
$\vec{E}$
$\vec{E}$
$v \approx c$