Homework 3
PH462 EM Waves and Optical Physics
due 14 Sept. 2007 in class
posted: 8 Sept 2007

Reading: Heald and Marion (HM) 5.1-5.5, posted notes

1) HM 5.2
2) A beam is polarized in the vertical (y) direction, and propagates in the z direction. It passes through a half wave plate which can be rotated around the z axis. Suppose the ordinary axis is at an angle $\theta$ to the x -axis.
a. Express the input state as a linear combination of linearly polarized basis vectors that are aligned with the crystal axes.
b. Using this representation of the input wave, apply the relative propagation phase shift to the wave that results from the half-wave plate.
c. Finally, express the output wave in terms of the original basis (the $x-y$ coordinate system) and show that the waveplate rotates linear polarization by $2 \theta$.

$x^{\prime}=$ ordinary axis ( $\mathrm{n}_{\mathrm{o}}$ )
$y^{\prime}=$ extraordinary axis $\left(n_{e}\right)$
3) An optically active medium has a refractive index that is different for right- and left-circularly polarized light ( $n_{R}$ and $n_{L}$ ).
a. Consider an input beam that is initially vertically polarized. It passes through an optically active medium through a distance $d$. Represent the input as a linear combination of R- and L-circular polarized state.
b. Apply the propagation phase for the two different polarization states to calculate the output polarization state. Show that the output polarization state is always linear, independent of $d$, but with a degree of rotation that linearly depends on $d$.
c. For crystal quartz, the specific rotation is 21.684 degrees $/ \mathrm{mm}$. Calculate the difference between $n_{R}$ and $n_{L}$ assuming a wavelength of 500 nm . Fixed
rotators made of quartz are often used in commercial optical systems instead of half-wave plates because they are insensitive to misalignment.
4) HM 5.7. In this problem, you are working out the basics of EM wave interference.
5) HM 5.9. In the first two parts, accounting for all the cosine factors is the main part. For a given ray that is reflected, the direction of the momentum shift of the light is normal to the surface. This problem can be done either by considering the momentum vectors and calculating the intensity on the surface, or by using the stress tensor. In the latter case, assume the E-field is aligned parallel to the surface, and the B-field is, as always, perpendicular to E and k. As mentioned in the text, there is a sign change for the reflected E-field so that there is no net Efield at the surface, and the B-fields add. For the later parts, you are integrating the pressure over the surface of the sphere.
6) As an application of the previous problem (part c), calculate the average power of a laser beam required to keep an absorbing sphere of $10^{-8} \mathrm{~g}$ and $20 \mu \mathrm{~m}$ in diameter floating in midair. Assume the beam is focused to the same diameter as the bead.
