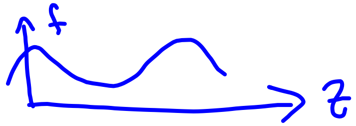
Wave Equation

1-D: $\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$

String



Show that any function $g(z-vt)$ is a solution to the wave eqn.

Define $u \equiv z - vt$

$g(u) = \sqrt{u},\ e^{iu},\ \cos(u)$

$\frac{\partial g}{\partial z} = \frac{\partial g}{\partial u}\frac{\partial u}{\partial z} = \frac{\partial g}{\partial u}$ ($\frac{\partial u}{\partial z} \to 1$)

$\frac{\partial}{\partial z}\left(\frac{\partial g}{\partial z}\right) = \frac{\partial^2 g}{\partial u^2}\frac{\partial u}{\partial z} = \frac{\partial^2 g}{\partial u^2}$

$\frac{\partial g}{\partial t} = \frac{\partial g}{\partial u}\frac{\partial u}{\partial t}$ ($\frac{\partial u}{\partial t} \to -v$)

$= -v\frac{\partial g}{\partial u} \Rightarrow \frac{\partial}{\partial t}\left(\frac{\partial g}{\partial t}\right) = -v\frac{\partial^2 g}{\partial u}\frac{\partial u}{\partial t} = v^2\frac{\partial^2 g}{\partial u^2}$

Plugging into wv. eqn.:

$\frac{\partial^2 g}{\partial u^2} = \frac{1}{v^2}\left(v^2\frac{\partial^2 g}{\partial u^2}\right)$ qed.

We'll concentrate on sinusoidal waves:

$$f(z,t) = A\cos(kz - \omega t + \delta)$$

$$v_{wave} = \frac{\omega}{k} \qquad |k| = \frac{2\pi}{\lambda}\ ; \quad \omega = 2\pi f$$

Euler's thm: $e^{ix} = \cos x + i\sin x$

$$f(z,t) = \mathrm{Re}\left[A e^{i[kz - \omega t + \delta]}\right]$$

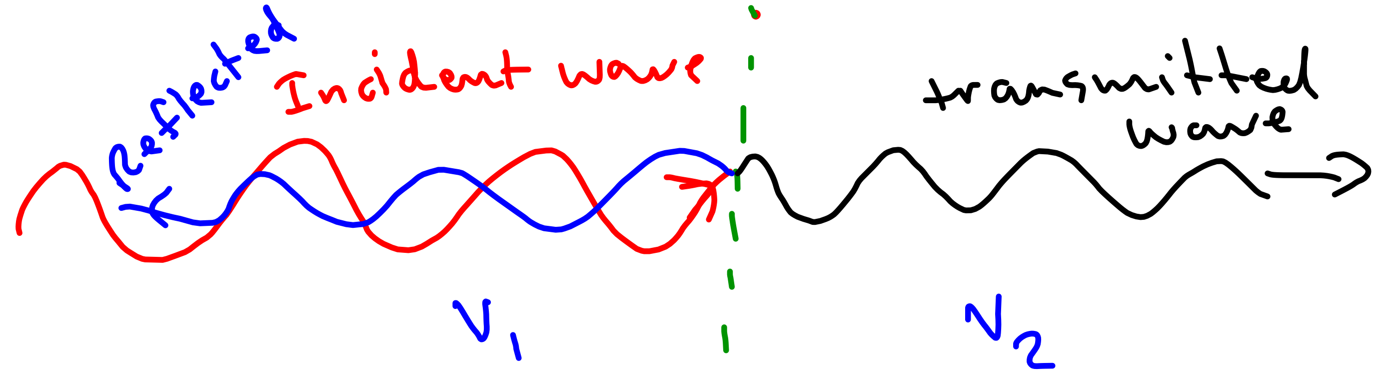
$$= \mathrm{Re}\Big[\underbrace{\underbrace{A e^{i\delta}}_{\tilde{A}}\, e^{i(kz-\omega t)}}_{\tilde{f}(z,t)}\Big]$$

$$\tilde{f}(z,t) = \tilde{A}\, e^{i(kz - \omega t)}$$

If I want $f_1 + f_2 = \mathrm{Re}[\tilde{f}_1] + \mathrm{Re}[\tilde{f}_2]$

$$= \mathrm{Re}[\tilde{f}_1 + \tilde{f}_2]$$

Any old function can be represented by $\tilde{f}(z,t) = \int_{-\infty}^{\infty} \tilde{A}(k)\, e^{i(kz - \omega t)}\, dk$



$\frac{\omega}{k_1} = v_1$

$\frac{\omega}{k_2} = v_2$

Incident: $\tilde{f}_I = \tilde{A}_I e^{i(k_1 z - \omega t)}$

$\tilde{f}_T = \tilde{A}_T e^{i(k_2 z - \omega t)}$

$\tilde{f}_R = \tilde{A}_R e^{i(-k_1 z - \omega t)}$

Boundary Cond: $\tilde{f}(0^-, t) = \tilde{f}(0^+, t)$

$\frac{\partial \tilde{f}}{\partial z}(0^-, t) = \frac{\partial \tilde{f}}{\partial z}(0^+, t)$

(1) Solve for $\tilde{A}_T$, $\tilde{A}_R$