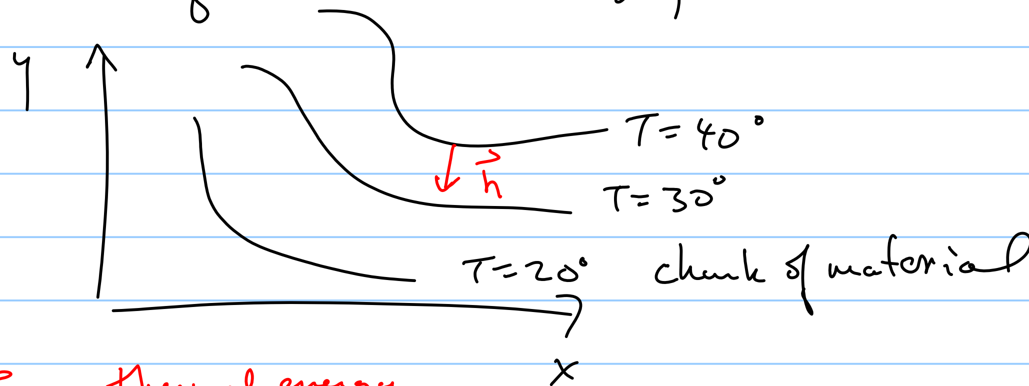
Lecture 14

Note Title 2/15/2006

Diverse solns to $\nabla^2 V = 0$ due to

Bndy conditions

Flow of Thermal energy



$\vec{h}$ is $\frac{\text{thermal energy}}{\text{area} \cdot \text{time}}$

temp flows "downhill" but $\vec{\nabla}$ points uphill

$$\vec{h} = -K \, \vec{\nabla} T(x,y,z,t)$$

↑ thermal conductivity

E & V analog

NOT A GENERAL RELATION (LIKE OHM'S law good only for certain materials) ASSUME NO SOURCES OF THERMAL Energy

$$\oint \vec{h} \cdot d\vec{a} = -\frac{dG}{dt}$$

↖ change in thermal energy inside vol

$$\int \vec{\nabla} \cdot \vec{h} \, d\tau = \oint \vec{h} \cdot d\vec{a}$$

$-\frac{dg}{dt}$ ← heat energy / vol

$$\vec{\nabla} \cdot \vec{h} = -\frac{dg}{dt}$$

$$g = c_v T$$

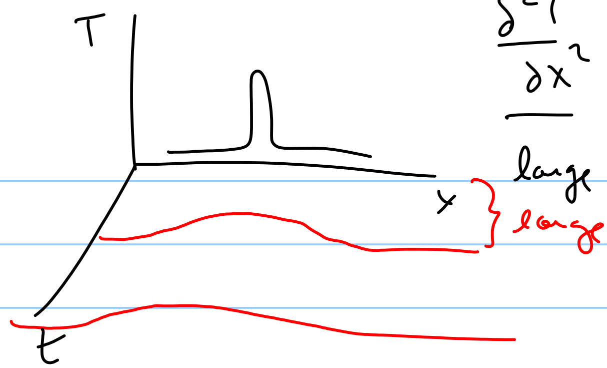
↖ heat capacity/vol

$$-K \vec{\nabla} \cdot \vec{\nabla} T = c_v \frac{\partial T}{\partial t}$$

$$\boxed{\nabla^2 T = -\frac{c_v}{K} \frac{\partial T}{\partial t}}$$

P.D.E.

1-D



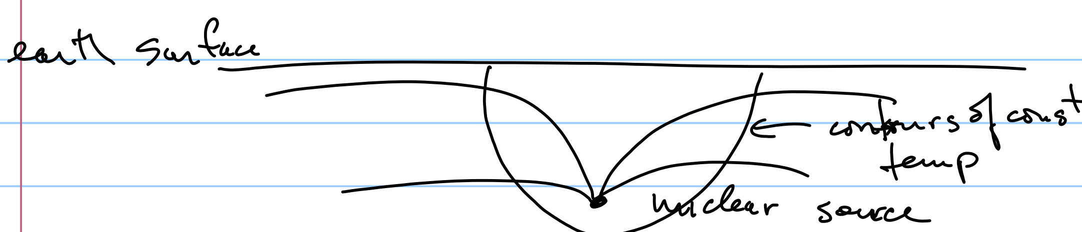
$$\frac{\partial^2 T}{\partial x^2} \propto \frac{\partial T}{\partial t}$$

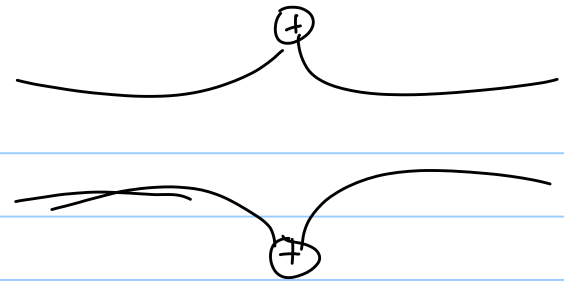
large initially — large

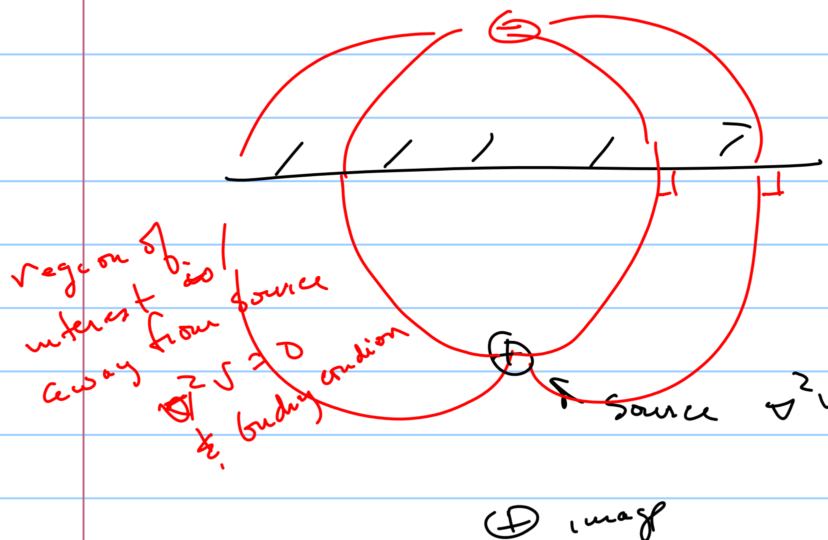
Thermostatics

$$\nabla^2 T = 0$$

Ex: Nuclear weapon ignited underground
(generates heat as a point source for a long time)
Bndy: Let no thermal energy escape from the surface of earth $\Rightarrow$ $h_\perp = 0$
· image







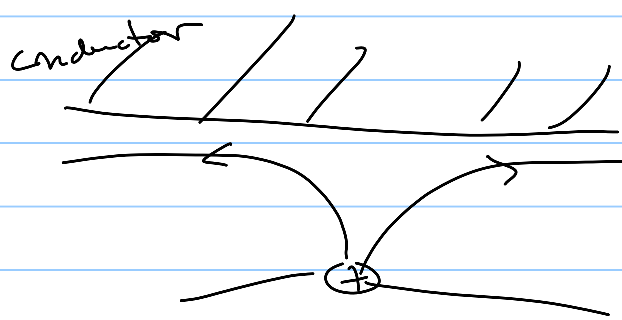
source $\nabla^2 V = -\rho/\epsilon_0$

boundary condition

$E_\perp \neq 0 = -\frac{\partial V}{\partial n}$

$E_\parallel = 0$

given $\frac{\partial V}{\partial n}$



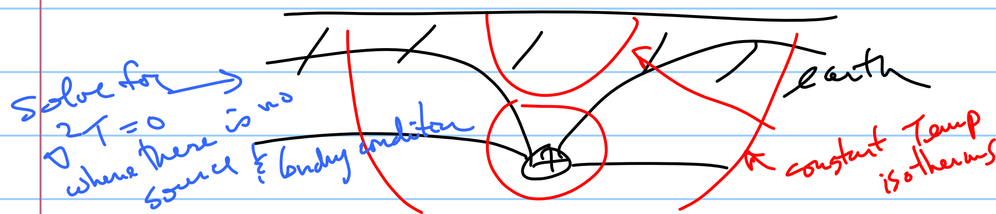
$E_\parallel \neq 0$

Bndry condition

- given $\frac{\partial V}{\partial n}$ or $E_\perp$ on surface
- given V on surface (sep. variables)

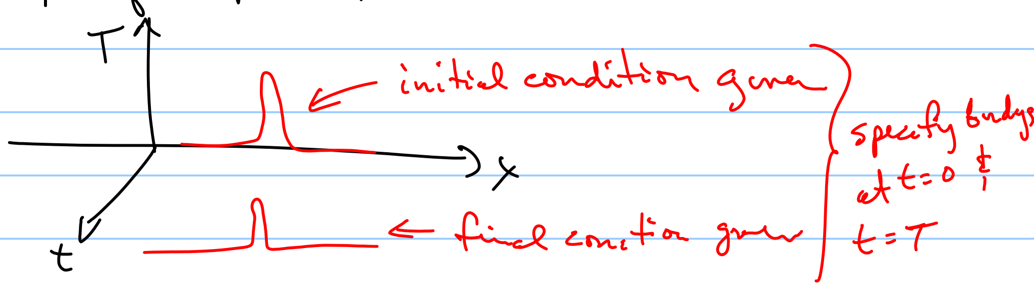
Nuclear problem given on bndry $h_\perp = 0$

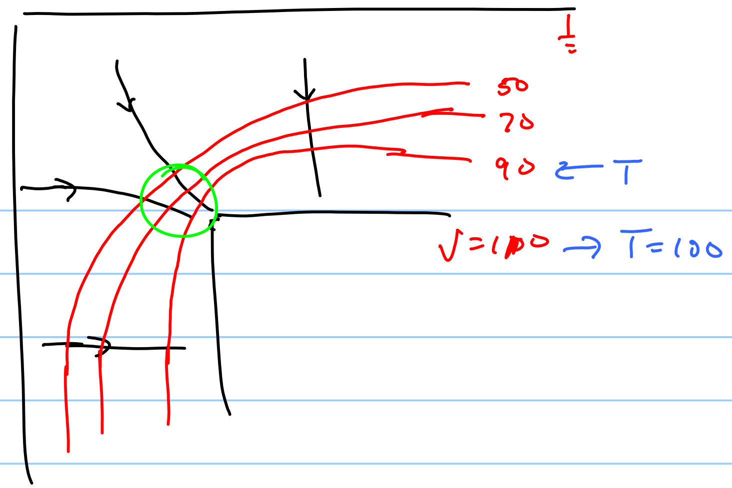
Soln image



Well posed problem

example of illposed problem





$$-i\hbar \frac{\partial \psi}{\partial t} = \frac{\hbar^2}{2m} \nabla^2 \psi$$ free particle Sch. eqn

↗ (diffusion eqn)