PHGN 462 Homework 7

1) Pollack and Stump 15.2. I like this one a lot. Ever since Phys 200 we've been talking about the field made by an infinite current-carrying wire, but we couldn't talk about the part where we actually turn on the current. Now we can.

2) Pollack and Stump 15.15.

3) The solutions to the differential equations for V and **A** in the Lorentz gauge (equations 15.5 and 15.6 in the book) are pretty clean as long as you evaluate the charge and current densities at the retarded time $t - {}^{r}/_{c}$. This represents the fact that influences from a charge or current travel at some speed c and take an amount of time ${}^{r}/_{c}$ to reach some observation point. None of this is super shocking to most people.

You know what *is* kind of shocking? The retarded potentials 15.19 and 15.20 don't only work with $t - {}^{r}/_{c}$. They also work with $t + {}^{r}/_{c}$, called the advanced time. This is an example of the fact that most physical laws are invariant with respect to time reversal – that is, *for the most part*, nature doesn't care which direction time flows; the laws of nature still work in either direction.

Oh, right, I'm supposed to ask you to actually do something. Prove that 15.19 and 15.20 are, in fact, still solutions to 15.5 and 15.6 even with the advanced time instead of the retarded time. And by that I mean prove that 15.19 solves 15.5, since doing 15.20 for 15.6 is the exact same process on a component by component basis.

(also see next page)

4) (Read *all* of the below)

First do Pollack and Stump 15.3. This innocent-looking problem actually puts a nice happy bow on all of classical electromagnetism. Maxwell's equations in differential form,

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

sometimes cause people to conclude odd things because they treat changing electric and magnetic fields as sources on an equal footing with charges and currents. But of course, those aren't *really* equal. Ultimately the changing electric and magnetic fields themselves came from charges and currents. Indeed, eventually all E and B fields come from charges and currents, period. The equations that this problem has you derive make that clearer: They're differential equations for E & B that explicitly lay out charges and currents as the ultimate sources of the fields.

Now, if you ever take graduate E&M, you may have the good fortune to solve these differential equations using Green's functions to get the following integral solutions:

$$\vec{E}(x,t) = \frac{1}{4\pi\varepsilon_0}\int \left\{ \frac{\vec{x}-\vec{x}'}{|\vec{x}-\vec{x}'|^3}\rho(\vec{x}',t') + \frac{\vec{x}-\vec{x}'}{c|\vec{x}-\vec{x}'|^2}\frac{\partial \rho(\vec{x}',t')}{\partial t'} - \frac{1}{c^2|\vec{x}-\vec{x}'|}\frac{\partial \vec{J}(\vec{x}',t')}{\partial t'} \right\} d^3x'$$

$$\vec{B}(x,t) = \frac{\mu_0}{4\pi}\int \left\{ \vec{J}(\vec{x}',t') \times \frac{\vec{x}-\vec{x}'}{|\vec{x}-\vec{x}'|^3} + \frac{\partial \vec{J}(\vec{x}',t')}{\partial t'} \times \frac{\vec{x}-\vec{x}'}{c|\vec{x}-\vec{x}'|^2} \right\} d^3x'$$

with everything in the integrand evaluated at the retarded time, as usual. These are known as Jefimenko's equations, and are the complete integral prescription for figuring out fields, given complete information about the charges and currents in the neighborhood. Hopefully you're still reading, because this is the place where I tell you that this problem is worth 15 points instead of 10, with the last 5 points coming from you commenting on how awesome this Jefimenko perspective is. Also examine the structure of the Jefimenko equations and comment on whether they make any intuitive sense and why (or why not).