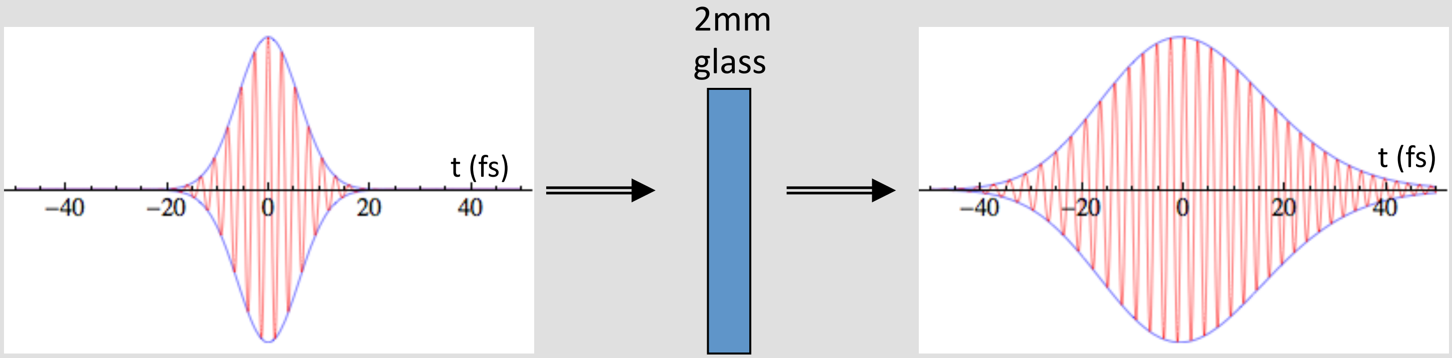
Pulse propagation: t/ω domain

- Dispersion in a system will stretch a short pulse:

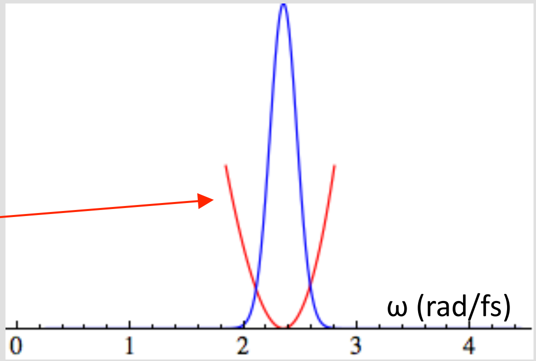


- Linear propagation is best represented in ω space:

$$E_{out}(\omega) = A(\omega - \omega_0)\, e^{i\phi(\omega)}$$

Spectral phase

$$\phi(\omega) = k\,L = \frac{\omega}{c}\, n(\omega)\, L$$

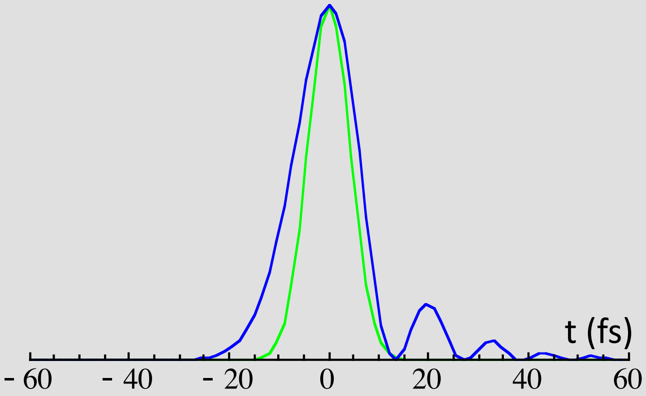


- Expand φ(ω) in series:

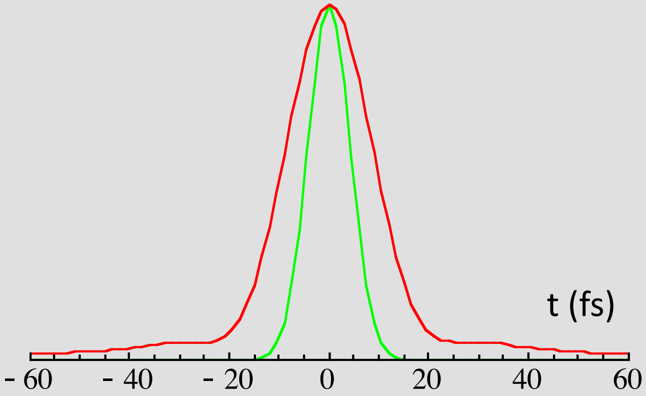
$$\phi(\omega) = \phi_0 + \phi_1(\omega - \omega_0) + \tfrac{1}{2}\phi_2(\omega - \omega_0)^2 + \tfrac{1}{3!}\phi_3(\omega - \omega_0)^3 + \tfrac{1}{4!}\phi_4(\omega - \omega_0)^4 + \cdots$$

Effects of residual high-order phase

- Compensate linear chirp, $\varphi_{2,}$ only (φ_3-limited) :



- Compensate φ_2 and φ_3 (φ_4-limited):



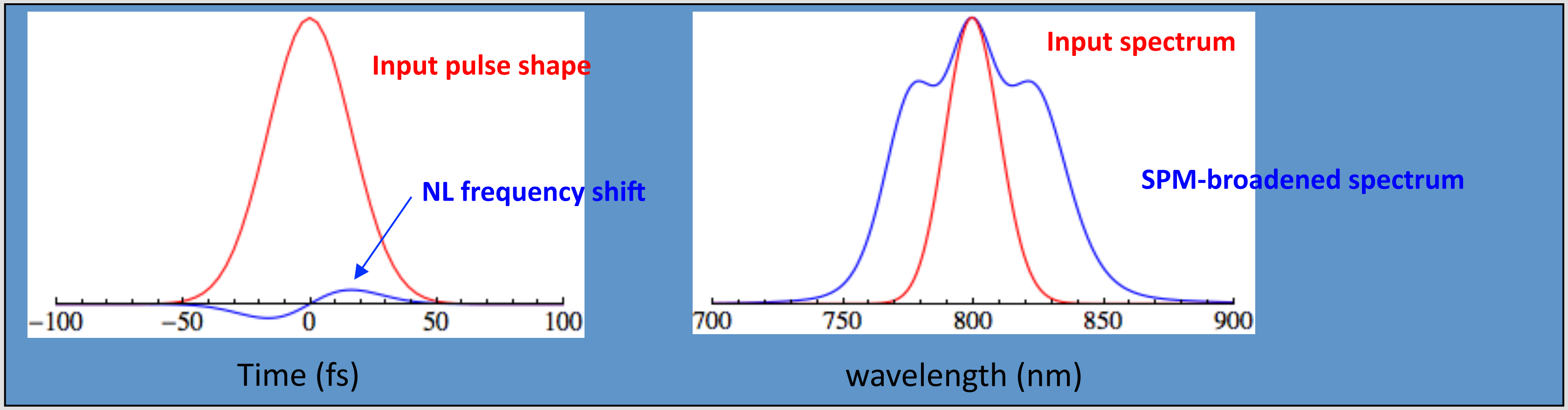
Nonlinear phase shifts: self-phase modulation

At high intensity, the refractive index can be changed noticeably:

$$-i\frac{dA}{dz} = \boxed{\underset{\text{SPM}}{\gamma_0 |A|^2 A}} + \beta_2 \frac{\partial^2 A}{\partial z^2}$$

Typically, relative sign of γ_0 and β_2 do not allow for solitons.

B-integral (NL phase shift): $B \approx \gamma_0 |A|^2 L \longrightarrow L_{NL} \approx 1/\gamma_0 |A|^2$



Dispersion length: $L_D \sim \frac{\tau_p^2}{\beta_2}$

Output spectrum and pulse shape are dramatically affected by dispersion.

Self-phase modulation & continuum generation

The self-phase-modulated pulse develops a phase vs. time proportional to the input pulse intensity vs. time.

Pulse intensity vs. time

$$E_0(z,t) = E_0(0,t)\exp\left[i\,k_0\,n_2\,I(t)\,z\right]$$

The further the pulse travels, the more modulation occurs.

That is: $\boxed{\phi(z,t) \approx k_0\,n_2\,I(t)\,z}$

A flat phase vs. time yields the narrowest spectrum. If we assume the pulse starts with a flat phase, then SPM broadens the spectrum.

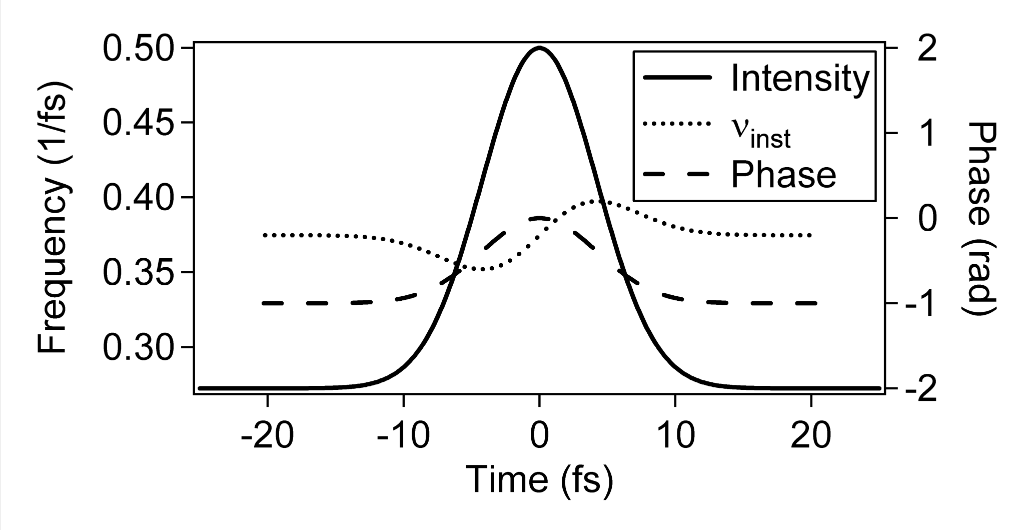
This is not a small effect! A total phase variation of hundreds can occur!
A broad spectrum generated in this manner is called Continuum.

The instantaneous frequency vs. time in SPM

$$\phi(z,t) \approx k_0 \, z \, n_2 \, I(t)$$

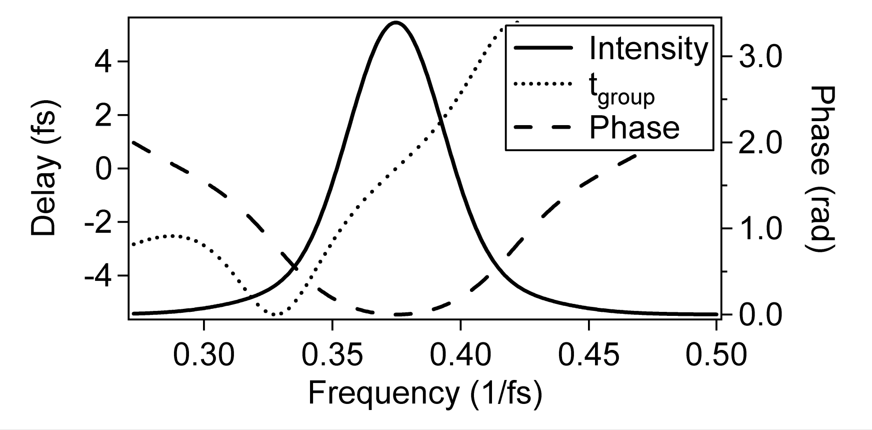
$$\omega_{inst}(t) = -\frac{\partial \phi(z,t)}{\partial t} = -k_0 \, z \, n_2 \, \frac{\partial I(t)}{\partial t}$$

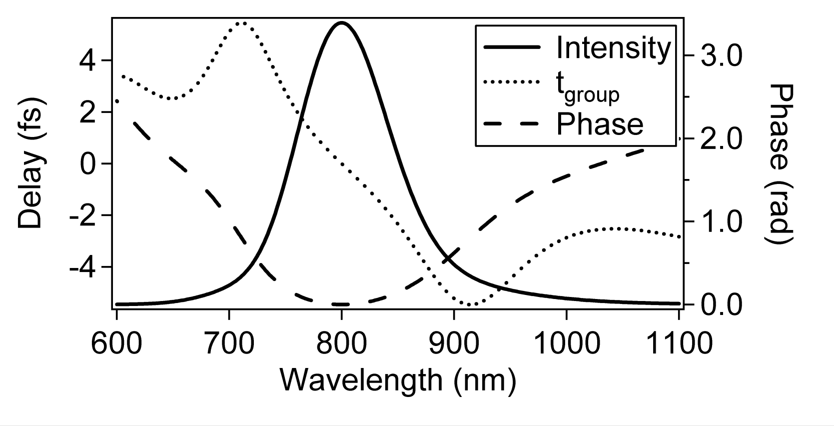
A 10-fs, 800-nm pulse that's experienced self-phase modulation with a peak magnitude of 1 radian.



Self-phase-modulated pulse in the frequency domain

The same 10-fs, 800-nm pulse that's experienced self-phase modulation with a peak magnitude of 1 radian.

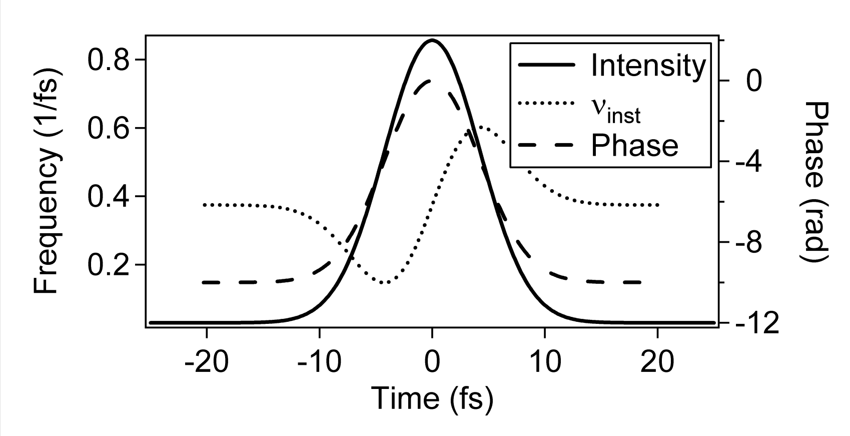


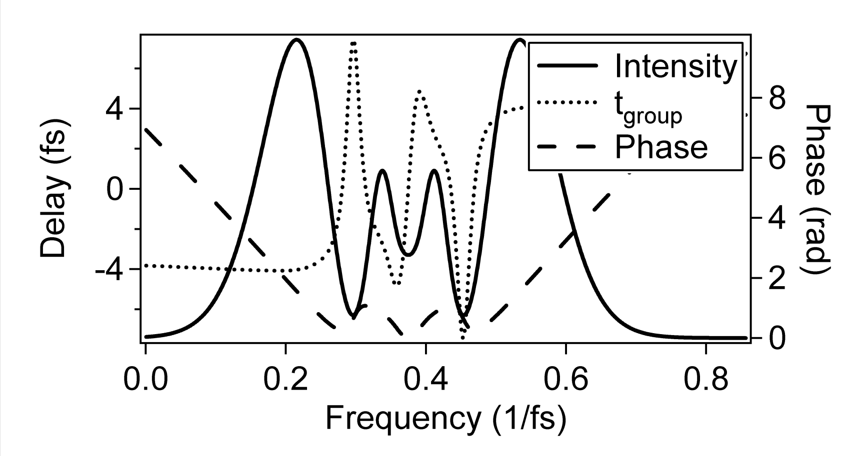


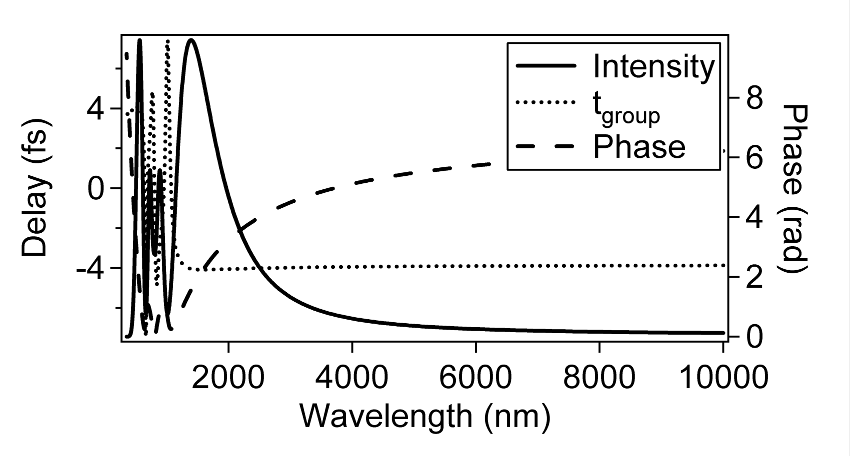
It' s easy to achieve many radians for phase delay, however.

A highly self-phase-modulated pulse

A 10-fs, 800-nm pulse that's experienced self-phase modulation with a peak magnitude of 10 radians







Note that the spectrum has broadened significantly. When SPM is very strong, it broadens the spectrum a lot. We call this effect continuum generation.

Experimental continuum spectrum in a fiber

Continua created by propagating 500-fs 625nm pulses through 30 cm of single-mode fiber.

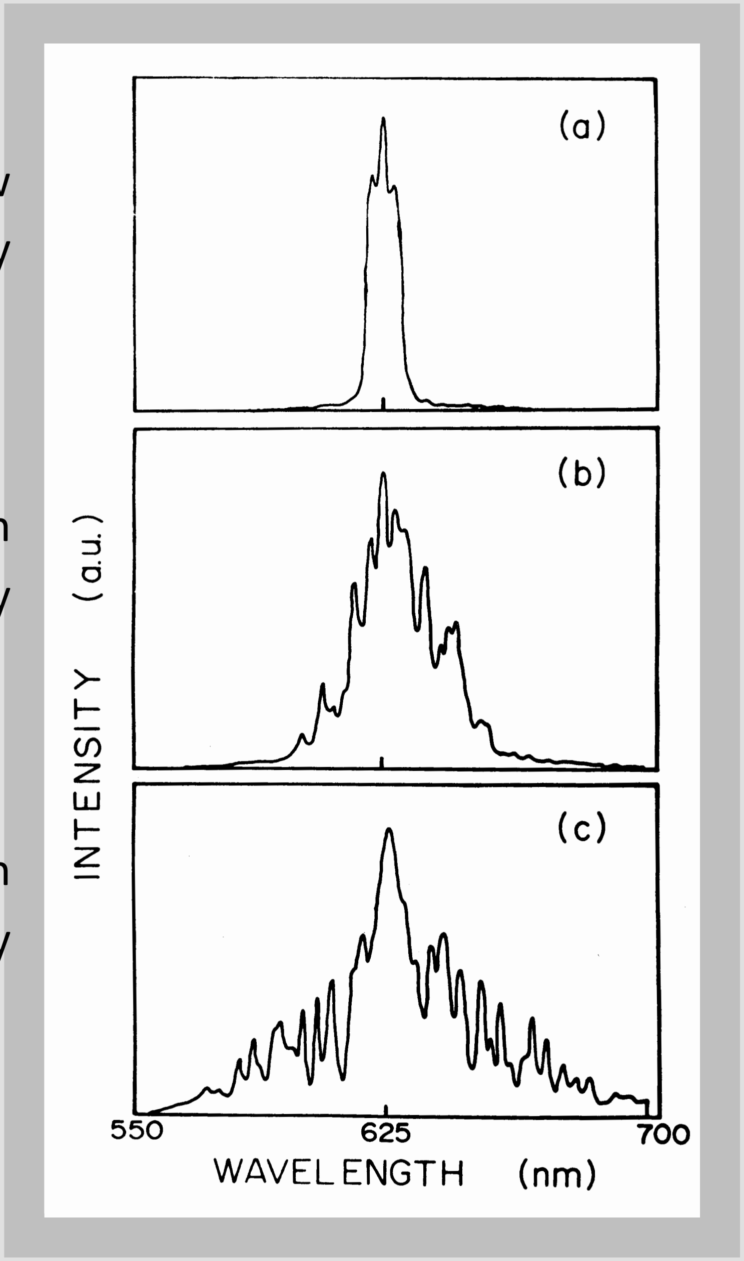
The Supercontinuum Laser Source, Alfano, ed.

Broadest spectrum occurs for highest energy.

Low Energy

Medium Energy

High Energy

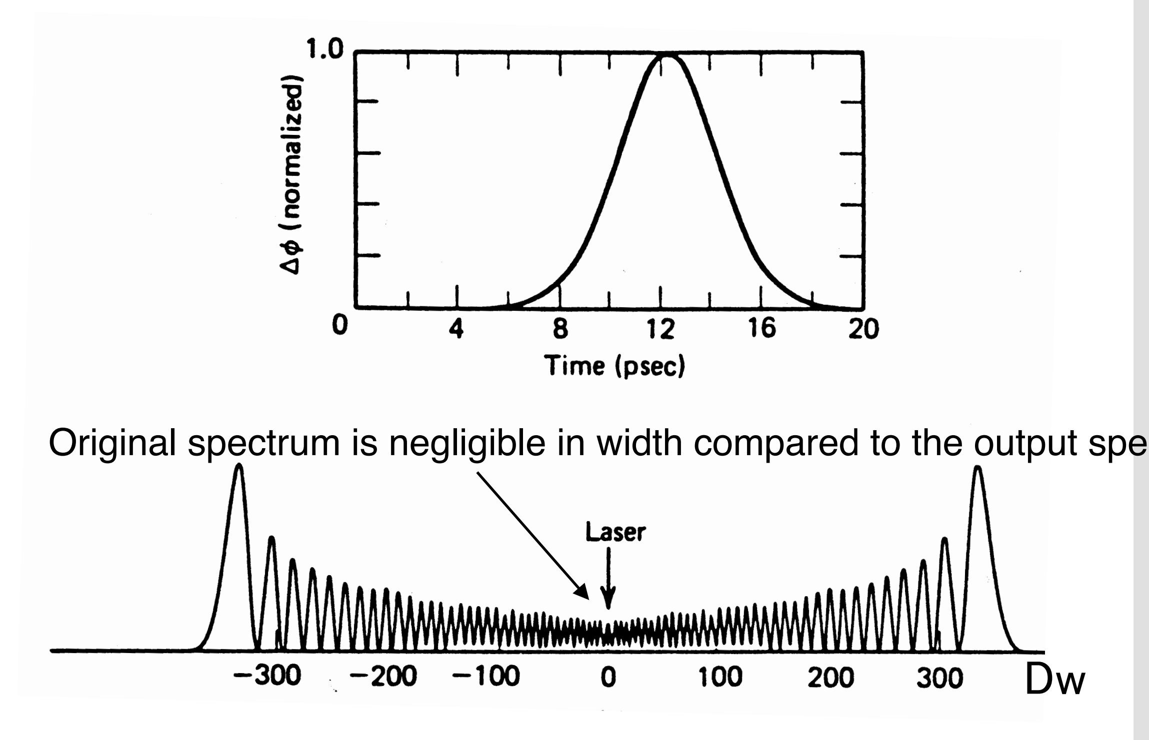


Continuum generation simulations

Instantaneously responding n_2; maximum SPM phase = 72π radians

Input Intensity vs. time (and hence output phase vs. time)

Output spectrum:



Original spectrum is negligible in width compared to the output spectrum.

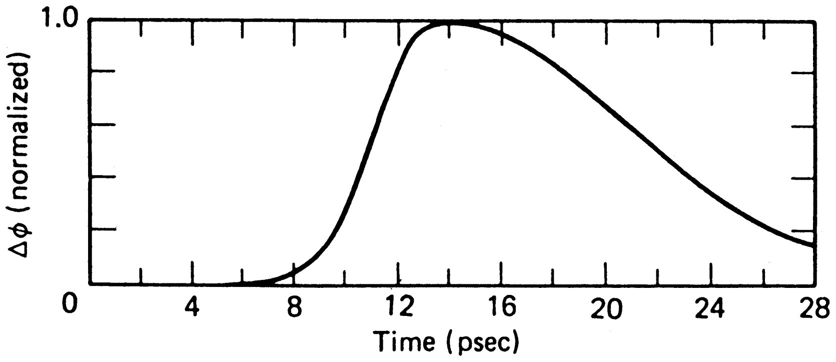
The Super-continuum Laser Source, Alfano, ed.

Oscillations occur in spectrum because all frequencies occur twice and interfere, except for inflection points, which yield maximum and minimum frequencies.

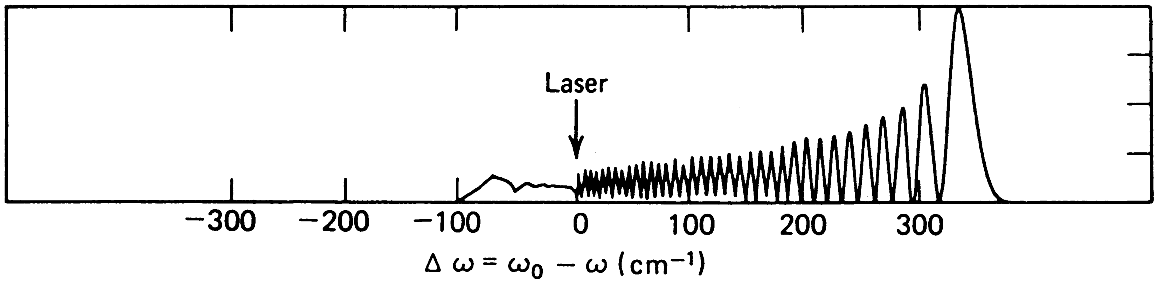
Continuum generation simulation

Noninstantaneously responding n_2; maximum SPM phase = 72π radians

Output phase vs. time (≠ input intensity vs. time, due to slow response)



Output spectrum:

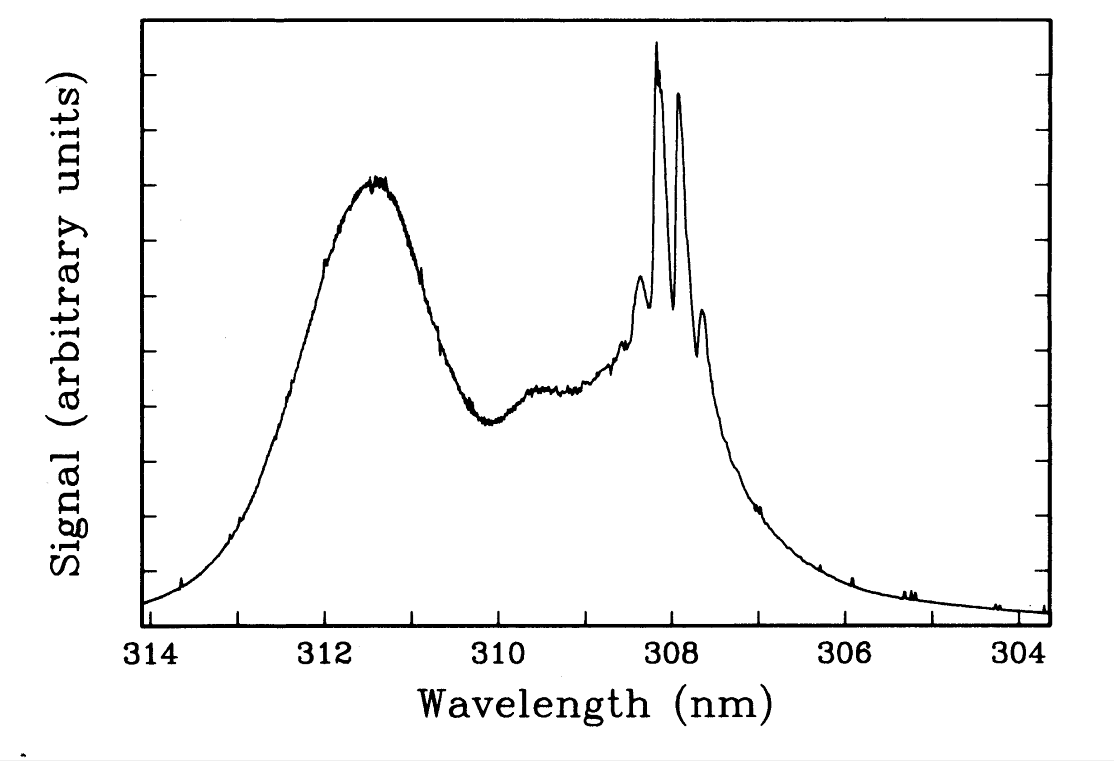


Asymmetry in phase vs. time yields asymmetry in spectrum.

The Supercontinuum Laser Source, Alfano, ed.

UV Continuum in Air!

308 nm input pulse; weak focusing with a 1m lens.

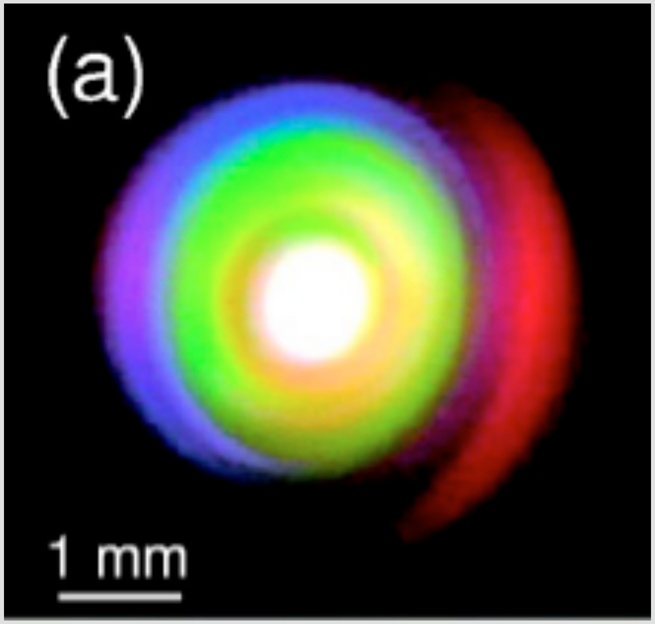


The Super-
continuum
Laser
Source,
Alfano, ed.

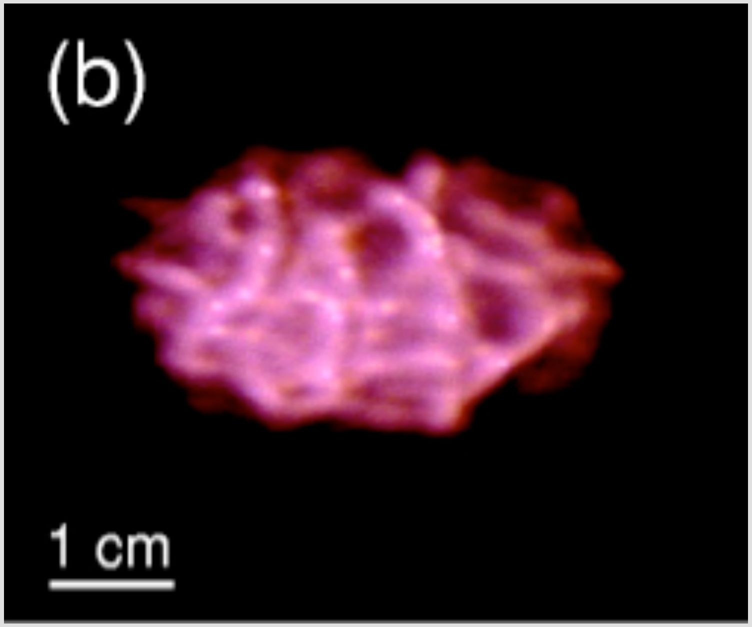
Continuum is limited when GVD causes the pulse to spread, reducing the intensity.

Continuum in air

Use a negatively chirped pulse to pre-compensate for the dispersion of air. Then “temporal focusing” occurs. The self-focusing can compensate diffraction: light bullets! Usually this only happens for pieces of the beam: filamentation, and it’s messy.



Conical emission from a fs beam in air, near the critical power P_{cr}.



Beam profile of a high-power beam (~1000P_{cr}) after 15m. Note the multiple filamentation.

Continuum Generation:
Good news and bad news

Good news:

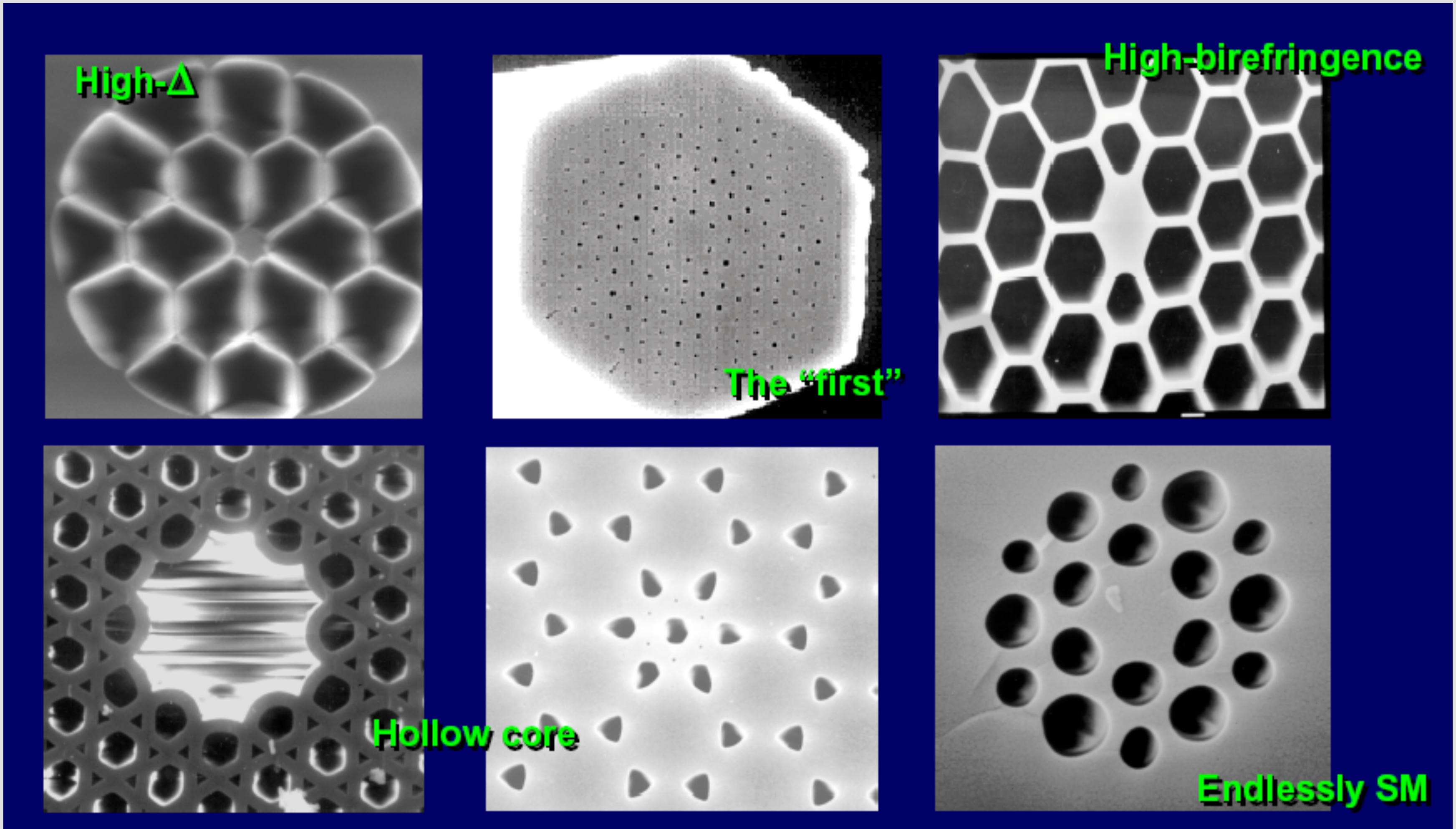
It broadens the spectrum, offering a useful ultrafast white-light source and possible pulse shortening.

Bad news:

Pulse shapes are uncontrollable.
Theory is struggling to keep up with experiments.
In a bulk medium, continuum can be high-energy, but it's a mess spatially.
In a fiber, continuum is clean, but it's low-energy.
In hollow fibers, things get somewhat better.

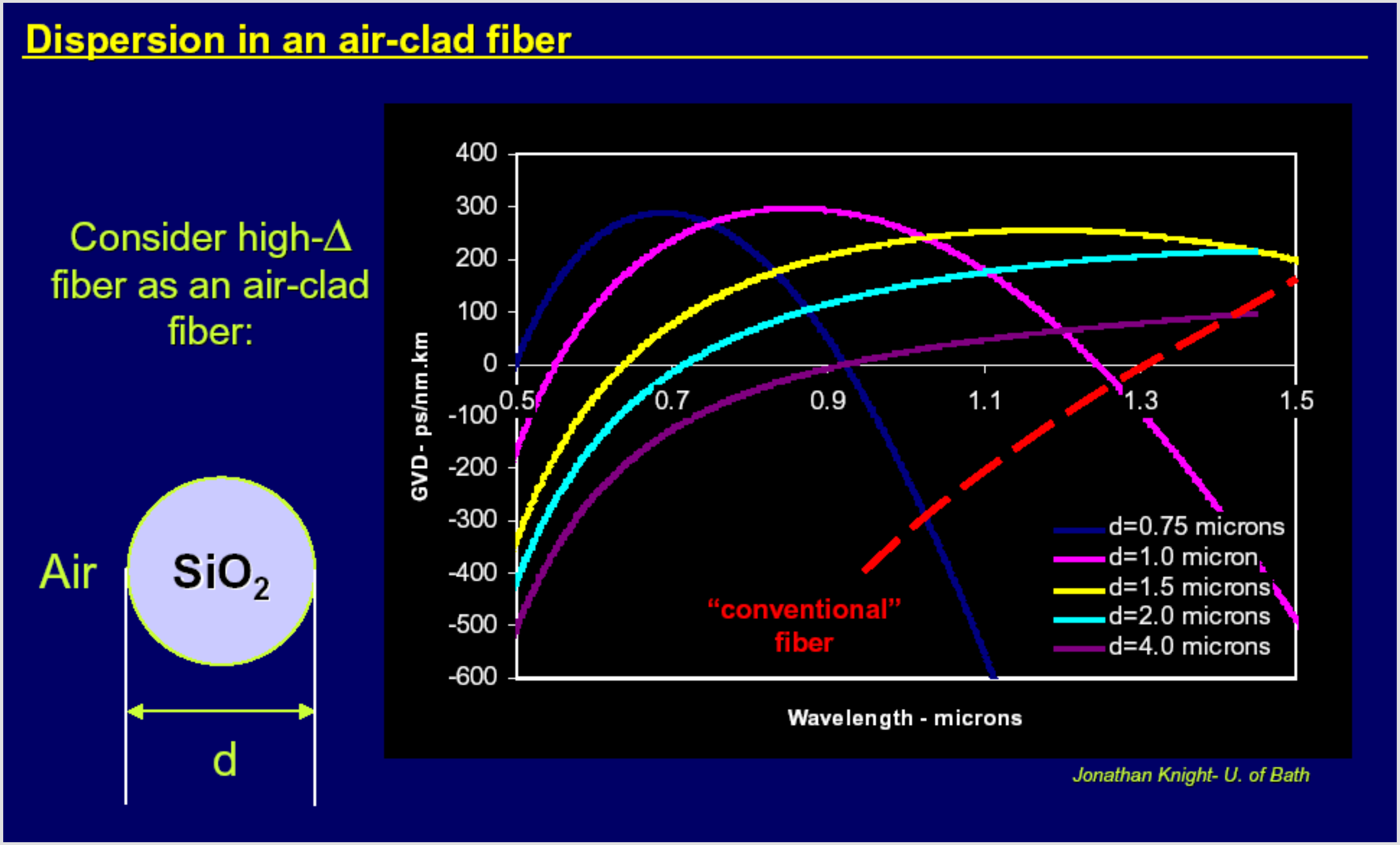
Main problem: dispersion spreads the pulse, limiting the spectral broadening.

Microstructure optical fiber



LANL / University of Bath

Microstructure optical fibers modify dispersion.

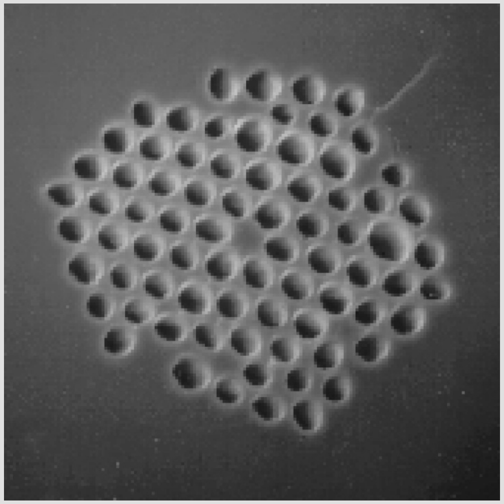


Dispersion in an air-clad fiber

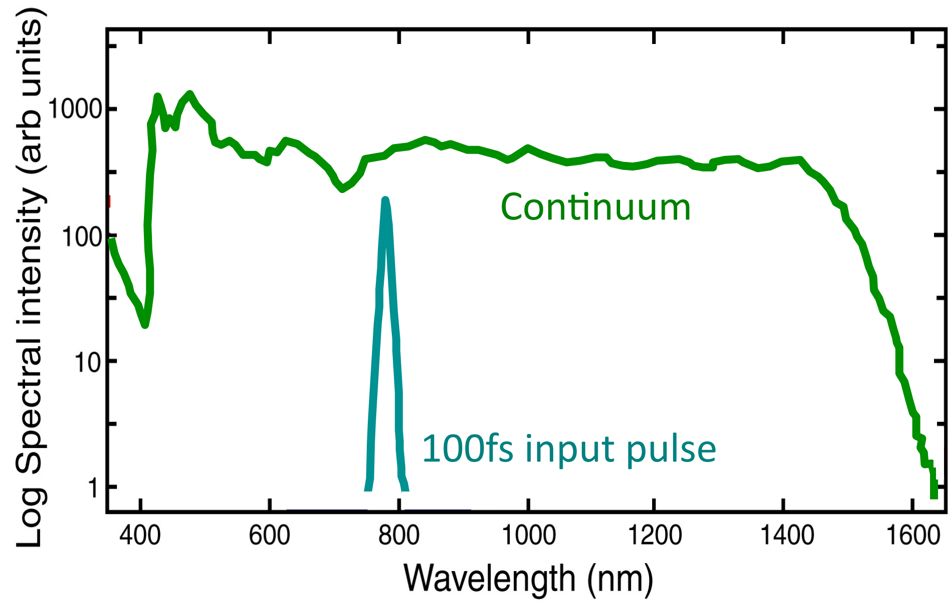
Consider high-Δ fiber as an air-clad fiber:

Jonathan Knight- U. of Bath

The continuum from microstructure optical fiber is ultrabroadband.



Cross section of the microstructure fiber.



- The spectrum extends from ~400 to ~1500 nm and is relatively flat (when averaged over time).

This continuum was created using *unamplified* Ti:Sapphire pulses.

J.K. Ranka, R.S. Windeler, and A.J. Stentz, Opt. Lett. Vol. 25, pp. 25-27, 2000

Continuum is quite beautiful!