

From the text:

17. In calculus, it can be shown that

$$\int \sin^2 x \, dx = \frac{x}{2} - \frac{1}{2} \sin x \cos x + C \text{ and}$$
$$\int \cos^2 x \, dx = \frac{x}{2} + \frac{1}{2} \sin x \cos x + C$$

Using integration by parts, it is also possible to prove that for each natural number n ,

$$\int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx \text{ and}$$
$$\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx$$

(a) Determine the values of

$$\int_0^{\pi/2} \sin^2 x \, dx \text{ and } \int_0^{\pi/2} \sin^4 x \, dx$$

(b) Use mathematical induction to prove that for each natural number n ,

$$\int_0^{\pi/2} \sin^{2n} x \, dx = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1) \pi}{2 \cdot 4 \cdot 6 \cdots (2n) \cdot 2}$$
$$\int_0^{\pi/2} \sin^{2n+1} x \, dx = \frac{2 \cdot 4 \cdot 6 \cdots (2n)}{1 \cdot 3 \cdot 5 \cdots (2n+1)}$$

These are known as the **Wallis sine formulas**.