MATH225, Fall 2008 **Name:**
Worksheet 5 (2.2-2.3, 3.1-3.2) **Section:**

For full credit, you must show all work and box answers.

1. Given

$$\begin{aligned} \frac{dx}{dt} &= y(x^2+y^2-1) \\ \frac{dy}{dt} &= -x(x^2+y^2-1) \end{aligned}$$

   (a) Is this system linear?

   (b) Find the equilibrium solutions for this system.

2. Given

$$\begin{aligned} \frac{dx}{dt} &= 2x+y^3 \\ \frac{dy}{dt} &= y \end{aligned}$$

   (a) Is this system linear?

   (b) Find the general solution to the system.

   (c) Find the particular solution that satisfies the initial condition $(x(0), y(0)) = (1, 1)$.

3. Given

$$\begin{aligned} \frac{dx}{dt} &= 2x + 3y \\ \frac{dy}{dt} &= x \end{aligned}$$

(a) Is this system linear?

(b) Rewrite the system in matrix-vector form.

(c) Are $\mathbf{Y}_1(t) = \begin{pmatrix} -e^{-t} \\ e^{-t} \end{pmatrix}$ and $\mathbf{Y}_2(t) = \begin{pmatrix} -e^{-t} \\ 2e^{-t} \end{pmatrix}$ solutions to this system?

(d) Are $\mathbf{Y}_1(t) = \begin{pmatrix} -e^{-t} \\ e^{-t} \end{pmatrix}$ and $\mathbf{Y}_3(t) = \begin{pmatrix} 12e^{3t} \\ 4e^{3t} \end{pmatrix}$ solutions to this system?

(e) Are $\mathbf{Y}_1(t)$ and $\mathbf{Y}_3(t)$ linearly independent?

(f) Find the general solution to the system. What principle are you using to do this? (Hint: You do not need to calculate eigenvalues and eigenvectors.)

4. Given

$$\begin{aligned} \frac{dx}{dt} &= 5x + 4y \\ \frac{dy}{dt} &= 9x \end{aligned}$$

(a) Is this system linear?

(b) Find the general solution to the system.

(c) Find the particular solution that satisfies the initial condition $(x(0), y(0)) = (2, 15)$. Write your solution as one vector.