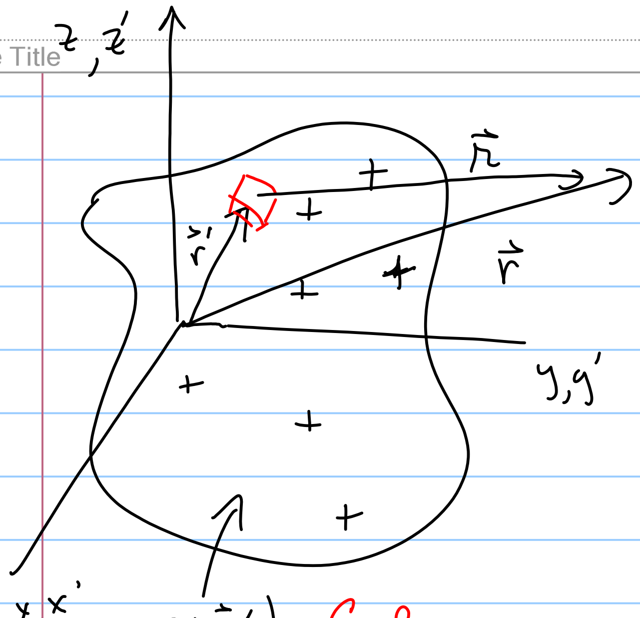
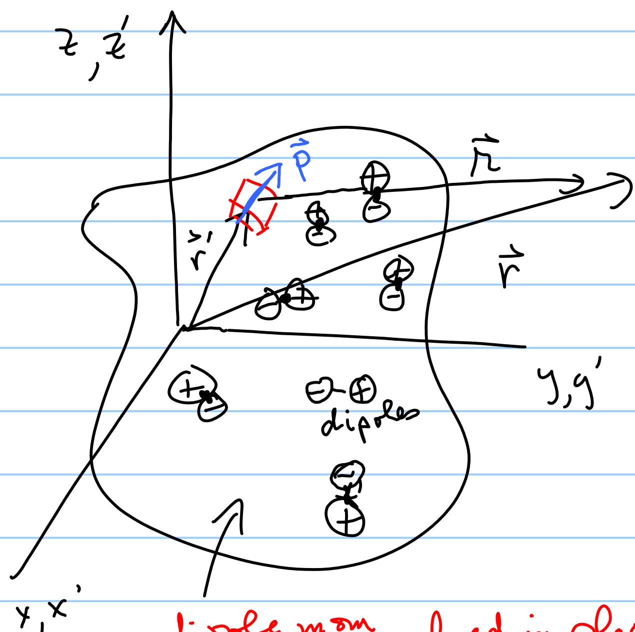
Note Title 2/19/2009



$\rho(\vec{r}\,')$ $\frac{\text{Coul}}{m^3}$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')\,d\tau'}{\mathscr{r}} + \frac{1}{4\pi\epsilon_0}\int \frac{\sigma\, da'}{\mathscr{r}}$$

$$V_{\text{pt charge}} = \frac{1}{4\pi\epsilon_0}\frac{q_i}{\mathscr{r}}$$



$\frac{\text{dipole mom}}{m^3}$ glued in place

$$V(\vec{r}) = ?$$

$$V_{\text{one dipole}} = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3}$$

$$V(\vec{r}) = \sum_i \frac{1}{4\pi\epsilon_0}\frac{\vec{p}_i\cdot\vec{\mathscr{r}}}{\mathscr{r}^3}$$

Dipole is NOT at origin but at $\vec{r}\,'$

charge/vol

For a continuous distribution of charge $q_i \rightarrow dq = \rho\, d\tau'$

For a continuous distribution of dipoles $\vec{p}_i \rightarrow d\vec{p} = \vec{P}\, d\tau'$

$\vec{P}$ is $\frac{\text{dipole moment}}{\text{vol}}$

$$V(\vec{r}) = \sum_i \frac{1}{4\pi\epsilon_0} \frac{\vec{P}_i \cdot \vec{\mathscr{r}}}{\mathscr{r}^3} \longrightarrow \int \frac{1}{4\pi\epsilon_0} \frac{\vec{P}\, d\tau' \cdot \vec{\mathscr{r}}}{\mathscr{r}^3}$$

$$dV = \frac{1}{4\pi\epsilon_0} \frac{\vec{P}\cdot(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\, d\tau' \qquad \vec{P}(\vec{r}\,')$$

Note $\vec{\nabla}' \frac{1}{|\vec{r}-\vec{r}\,'|} = \frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3}$ & $\vec{\nabla} \frac{1}{|\vec{r}-\vec{r}\,'|} = -\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$

$$\vec{\nabla}' \frac{1}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}} = \hat{x}\frac{\partial}{\partial x'}\frac{1}{\sqrt{}} + \hat{y}\frac{\partial}{\partial y'}\frac{1}{\sqrt{}} + \hat{z}\frac{\partial}{\partial z'}\frac{1}{\sqrt{}}$$

$$\vec{\nabla}\cdot f\vec{A} = f\vec{\nabla}\cdot\vec{A} + \vec{A}\cdot\vec{\nabla}f$$

$$\vec{A}\cdot\vec{\nabla}f = \vec{\nabla}\cdot f\vec{A} - f\vec{\nabla}\cdot\vec{A}$$

$$V = \frac{1}{4\pi\epsilon_0}\int \vec{P}\cdot\vec{\nabla}'\left(\frac{1}{|\vec{r}-\vec{r}\,'|}\right) d\tau'$$

$$\vec{A}\cdot\vec{\nabla}(f) = \vec{\nabla}\cdot f\vec{A} - f\,\vec{\nabla}\cdot\vec{A}$$

$$V = \frac{1}{4\pi\epsilon_0}\left[\int \vec{\nabla}'\cdot\left[\frac{\vec{P}}{|\vec{r}-\vec{r}\,'|}\right] d\tau' - \int \frac{1}{|\vec{r}-\vec{r}\,'|}\vec{\nabla}'\cdot\vec{P}\, d\tau'\right]$$

DIVERGENCE THEOREM = ?

Does this look like the voltage from any charge distribution you have seen?

$$\vec{\nabla}\cdot f\vec{A} = f\vec{\nabla}\cdot\vec{A} + \vec{A}\cdot\vec{\nabla}f$$

$$\vec{A}\cdot\vec{\nabla}f = \vec{\nabla}\cdot f\vec{A} - f\vec{\nabla}\cdot\vec{A}$$

$$V = \frac{1}{4\pi\epsilon_0}\int \vec{P}\cdot\vec{\nabla}'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) d\tau'$$

$$\vec{A}\cdot\vec{\nabla}(f) = \vec{\nabla}\cdot f\vec{A} - f\vec{\nabla}\cdot\vec{A}$$

$$V = \frac{1}{4\pi\epsilon_0}\left[\int \vec{\nabla}'\cdot\frac{\vec{P}}{|\vec{r}-\vec{r}'|}d\tau' - \int \frac{1}{|\vec{r}-\vec{r}'|}\vec{\nabla}'\cdot\vec{P}\,d\tau'\right]$$

DIVERGENCE THEOREM

$$= \frac{1}{4\pi\epsilon_0}\oint \frac{\vec{P}\cdot d\vec{a}'}{|\vec{r}-\vec{r}'|} + \frac{1}{4\pi\epsilon_0}\int \frac{-\vec{\nabla}\cdot\vec{P}}{|\vec{r}-\vec{r}'|}d\tau'$$

over surface of material

$d\vec{a}' = \hat{n}\,da$ where $\hat{n}$ is $\perp$ to $d\vec{a}$

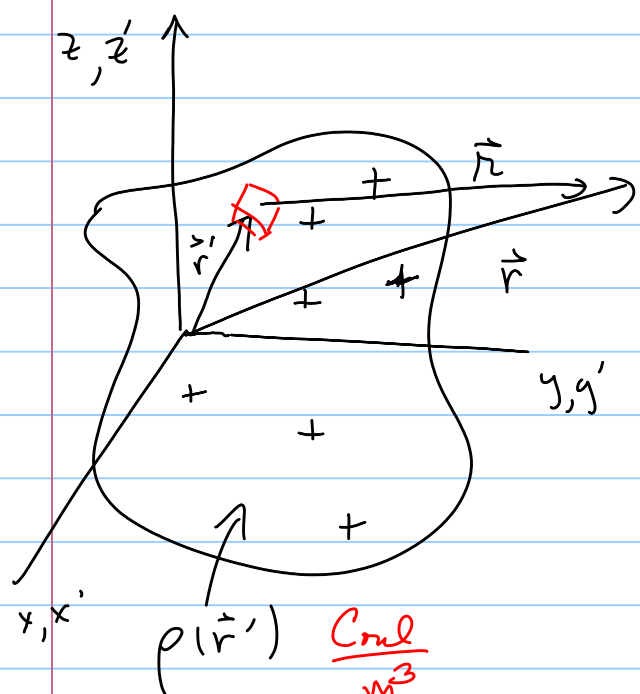
unit vector $\perp$ surface

$$d\vec{a} = \hat{n}\,da$$

$$= \frac{1}{4\pi\epsilon_0} \oint \frac{\sigma_b \, da}{|\vec{r} - \vec{r}\,'|} + \frac{1}{4\pi\epsilon_0} \int \frac{\rho_b \, d\tau'}{|\vec{r} - \vec{r}\,'|}$$

where $\sigma_b = \vec{P} \cdot \hat{n}$ $\qquad \rho_b = -\vec{\nabla} \cdot \vec{P}$

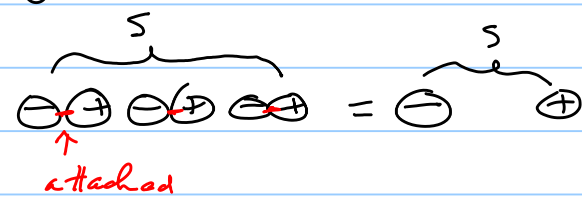
Compare with the result at the beginning of this lecture



$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}\,') \, d\tau'}{\mathscr{r}} + \frac{1}{4\pi\epsilon_0} \int \frac{\sigma \, d\tau'}{\mathscr{r}}$$

$$V_{\text{pt charge}} = \frac{1}{4\pi\epsilon_0} \frac{q_i}{\mathscr{r}}$$

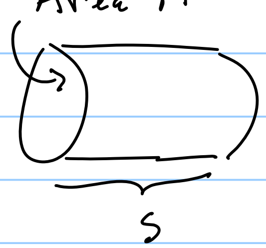
Physical Interpretation of Bound Charge

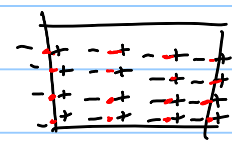
Consider a string of dipoles 

The head of one "cancels" the tail of its neighbor except at the ends where charge is left over.

Net charge at ends is the bound charge.
Assume uniform polarization or $\vec{P}$ = constant

Area A



$= \; -q \underbrace{\qquad\qquad}_{s} +q \; =$ 

This is equivalent to dipole moment $qs = PAs$

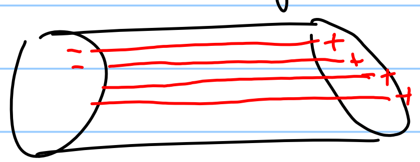
since $\vec{P}_{total} = \sum_i \vec{p}_i = \vec{P}\, \overbrace{As}^{vol}$

atom dipole moments — dipole mom/vol

so $q = PA$

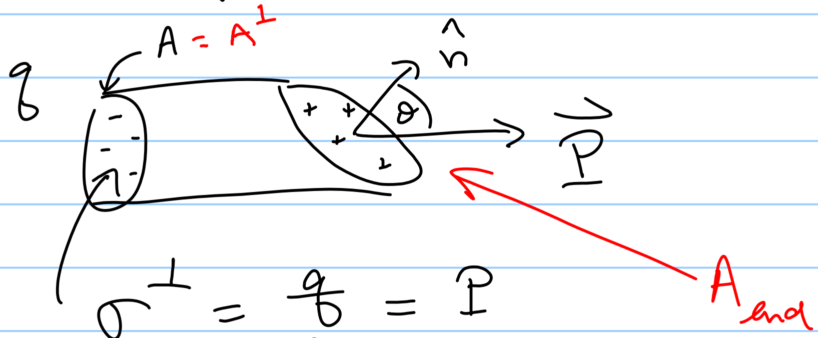
The surface charge density is $\frac{q}{A} = P$

For an oblique cut the charge is still the same



Think of each charge at the end of a fiber through the material. The number of fibers at the end is the same no matter what the cut.

If charge is same and area increases then σ decreases.



$$\sigma_b^{\perp} = \frac{q}{A} = P$$

$A^{\perp} = A = A_{end} \cos\theta$ so $\sigma_b = \frac{q}{A_{end}}$

$$\sigma_b = \frac{q}{A^{\perp}/\cos\theta} = \frac{q}{A^{\perp}} \cos\theta = \sigma_b^{\perp} \cos\theta = \vec{P} \cdot \hat{n} \quad \text{AT SURFACE}$$