$\swarrow$ radius R

2. (12 points) The potential at the surface of a sphere is $V(\theta) = V_0 \cos\theta$. Determine an integral expression for the voltage outside the sphere . The general radial solution is $R(r) = A_l r^l + \frac{B_l}{r^{l+1}}$ while the general angular solution is $P_l(\cos\theta)$ where l is an integer. Note: $\int_{-1}^{1} P_l(x)P_m(x)dx = \frac{2}{2l+1}\delta_{lm}$.

Separation of varibles soln doesn't satisfy bndry condition. Use superposition principle & orthogonality of $P_\ell(\cos\theta)$ to construct a general solution. Outside the sphere

$A_\ell r^\ell \to \infty$ as $r \to \infty$ so $A_\ell = 0$

$$V(r,\theta) = \sum_{\ell=0}^{\infty} \frac{B_\ell}{r^{\ell+1}} P_\ell(\cos\theta)$$ at bndry $V_0 \cos\theta = \sum \frac{B_\ell}{R^{\ell+1}} P_\ell(\cos\theta)$

Multiply both sides by $P_m(\cos\theta)$ & integrate to use orthogonality

$$\int_{-1}^{1} V_0 P_m(x)\, x\, dx = \sum \frac{B_\ell}{R^{\ell+1}} \underbrace{\int_{-1}^{1} P_m(x) P_\ell(x)\, dx}_{\frac{2}{2\ell+1}\delta_{\ell m}} = \frac{B_m}{R^{m+1}} \frac{2}{2m+1}$$

$$B_m = \frac{R^{m+1}(2m+1)}{2} \int_{-1}^{1} V_0 P(x)\, dx$$