MATH-332: Linear Algebra Chapter: 01

Applications of Linear Systems

Section 1.8: Introduction to Linear Transformations

pgs. 73-82 *June 22, 2009*

Lecture: Introduction to Linear Transformations

Topics:
Matrix Transformations
Linear Transformations
Domain, Co-domain, Range
one to one; onto - From 1.9 page 87
Geometric Linear Transformsions of $\mathbb{R}^2$ - From 1.9 page 85-87

Problems:
Prac: 1-3
Prob: 17, 19, 25, 31

This is another progressive section of Lay's textbook. Often one would postpone/dis-include thinking about $\mathbf{Ax} = \mathbf{b}$ as a function on the vector $\mathbf{x}$ that takes $\mathbf{x}$ to $\mathbf{b}$. However, it seems as if the author is a believer of the old idiom, 'in for a penny, in for a pound.' That is, if we have already inundated ourselves with so much language, then what's a little more? What we learn here is to re-contextualize our previous understanding of linear systems in terms of mappings of vector-variables between spaces. This concept is handy when trying to think about how a matrix *acts* on a vector both geometrically and algebraically and when this *action* can be undone.

Section Goals

- Understand how $\mathbf{Ax} = \mathbf{b}$ can be thought of as a linear function $\mathbf{A} : \mathbf{R}^m \to \mathbf{R}^n$ and how this mapping is related to the solubility of the linear system.
- Characterize mappings in terms of the sets which they map from and to. Relate these ideas to the linear mappings defined by matrix equations.

Section Objectives

- Define transformations and their associated language. That is, define linear transformation, domain, co-domain, range, one-to-one, onto.
- Consider explicit examples of linear transformations on $\mathbb{R}^2$ and highlight geometric properties of these transformations.