MATH-300 - Foundations of Adv. Mathematics **Field 2009**
Midterm 1 **Due: Thursday May 28, 5:00pm**

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1. In class, we proved the Triangle Inequality using an observation regarding absolute value. This time, however, we are going to take the long way to the proof. Using cases, prove

   For all real numbers x and y, $|x+y| \leq |x| + |y|$

2. Define A as the average of the n numbers, x_1, x_2, ..., x_n. Prove that at least one of the x_1,...,x_n is greater than or equal to A.

3. Let a and b be integers with $a \neq 0$. If a does not divide b, then the equation $ax^3 + bx + (b+a) = 0$ does not have a solution that is a natural number.
   (Hint: It may be necessary to factor a sum of cubes. Recall that $u^3 + v^3 = (u+v)(u^2 - uv + v^2)$.)

4. Prove the following proposition
   For all sets A, B, and C that are subsets of some universal set, if

   $$A \cap B = A \cap C \text{ and } A^c \cap B = A^c \cap C, \text{ then } B = C.$$