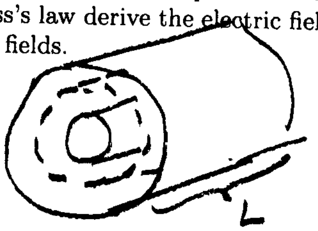
$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}$$

3. A cylindrical capacitor of capacitance C, and inner and outer radii a and b, is charged to voltage V. Using Gauss's law derive the electric field in the region between a and b in terms of C and V. Assume no fringing fields.



$\vec{E}$ is radial so choose cylinder for surface

$\vec{E}\cdot d\vec{a} = 0$ on both end caps. $\vec{E} \perp d\vec{a}$ for all tiles on body of cylinder so $\int_{body} \vec{E}\cdot d\vec{a} = \int E\,da$ ← $\vec{E}$ is the same for all tiles $\quad \int da = 2\pi r L$

$$E\int da = E\,2\pi r L = \frac{Q}{\epsilon_0} \Rightarrow E = \frac{Q}{2\pi\epsilon_0 r L} \quad \text{but } C = \frac{Q}{V}$$

So $Q = CV$

Ⓘ $q E_z(x,y,z) = m\frac{d^2 z}{dt^2}$ Ⓘⓘ $q E_x(x,y,z) = m\frac{d^2 x}{dt^2}$ Ⓘⓘⓘ $q E_y(x,y,z) = m\frac{d^2 y}{dt^2}$

Three <u>coupled</u> ODE's with bndry conditions $v_{0x}, v_{0y}, \text{ \& } v_{0z}$ and $r_{0x}, r_{0y}, \text{ \& } r_{0z}$

4. The charge configuration shown creates an electrostatic lens for an electron microscope. How would you determine the trajectory of an electron with initial velocity $\vec{v}_0$ and position $\vec{r}_0$?

Newtons Law $\quad \vec{F} = q\vec{E}(x,y,z) = m\frac{d\vec{v}(x,y,z,t)}{dt} \quad x(t)\ y(t)\ z(t)$

$$\vec{v} = v_x(t)\hat{x} + v_y(t)\hat{y} + v_z(t)\hat{z}$$

Newtons law is 3 eqns. Solve using initial velocity and do again to get trajectory. This reduces to

$$\left.\begin{aligned} qE_x(x,y,z) &= m\frac{dv_x(t)}{dt} \\ qE_y(x,y,z) &= m\frac{dv_y(t)}{dt} \\ qE_z(x,y,z) &= m\frac{dv_z(t)}{dt} \end{aligned}\right\} \text{3 1st order ODE's}$$

Compare with the example $F = -kx = m\frac{d^2x}{dt^2}$

$x = A\cos(\Omega t + \alpha)$ where A & α are determined by initial conditions.

OR solve numerically as discussed in class (see lecture notes)

SOLN is not $\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t - \frac{1}{2}at^2\hat{a}$