Supplement: Recent questions regarding vectors & pseudovectors
(or polar vectors & axial vectors)

Quick reminder: The nature of a vector is determined by how it reacts to parity operations, or inversions. In a coordinate inversion (or parity transformation), each coordinate transforms into its negative:

$$x \to -x, \quad y \to -y, \quad z \to -z$$

So your typical position vector goes: $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \to$

$$-x\hat{\imath} - y\hat{\jmath} - z\hat{k} = -\vec{r}$$

A normal vector, or a polar vector, is one such that $\vec{v} \to -\vec{v}$ under inversion. Position, velocity, and acceleration all fall under this category.

But consider certain vectors defined in terms of other vectors, like angular momentum

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$$

Under parity, $\vec{r} \to -\vec{r}$, $\vec{v} \to -\vec{v}$, so $\vec{L} \to -\vec{r} \times -\vec{p} = \vec{L}$

$\vec{L}$ <u>doesn't</u> change sign under parity. It's a pseudovector, or axial vector. And that one little difference in transformation behavior turns out to be fantastically important in particle physics, among other places.

$\boxed{\text{Question I}}$ (as I recall it)

Does the form of the Lorentz force law come from the fact that $\vec{B}$ is a pseudovector?

Well, since $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} = m\vec{a}$ and $\vec{a}$ is a vector, so must be $\vec{F}$, $\vec{E}$, and $\vec{v} \times \vec{B}$.

(Much as you only add scalars to scalars and vectors to other vectors, you should only add pseudovectors to pseudovectors. You can't add a vector to a pseudovector unless you give them a couple of drinks first)

So if $\vec{v} \times \vec{B}$ is a vector, under inversion it must transform like

$\vec{v} \times \vec{B} \to -(\vec{v} \times \vec{B})$ And $\vec{v}$ transforms to $-\vec{v}$, so for all this to work out it must be that $\vec{B} \to \vec{B}$ and is a pseudovector.

(more →)

So let me reframe the question a bit: Is it the fact that $\vec{B}$ is a pseudovector that forces the structure of $q\vec{v}\times\vec{B}$, or is it the fact that magnetic force goes like $q\vec{v}\times\vec{B}$ that forces us to conclude that $\vec{B}$ is a pseudovector? I favor the latter interpretation, since as we'll discover later, things like $q\vec{v}\times\vec{B}$ appear naturally when you go to make the behavior of $\vec{E}$ consistent with relativity.

A field theorist like Dr. Flournoy might have a more fundamental point of view that changes his answer.

Question II

When making those Ampere's Law symmetry arguments about flipping over a solenoid, have we disrespected the fact that $\vec{B}$ is a pseudovector?

Let's use a straight wire instead since it's a similar case physically but much easier to diagram. My claim was that it can't make a radial field because flipping the wire around wouldn't convert $\hat{r}$ fields to $-\hat{r}$, but according to the Biot-Savart law changing the sign of $I d\vec{l}$ (an equivalent operation) would, so we have a contradiction and the $\hat{r}$ term can't exist.

The counterargument is that under an inversion, $\vec{B}$ should keep the same sign since it's a pseudovector, so there's no problem and we can't rule out the $\hat{r}$ component.

Having pondered this some more I'm fairly convinced that the resolution comes from the wire flipping not being an inversion. When we rotate the wire 180° we go like this:

But in an inversion everything changes sign, including the $\vec{r}$ that represents where the observer is, so a proper inversion would look like:

Put another way, in an inversion we'd change $I d\vec{l} \to -I d\vec{l}$ and $\vec{r} \to -\vec{r}$ and expect from the Biot Savart law for the field to stay the same, consistent with $\vec{B}$'s pseudovectory nature. But in the 180° rotation we're performing, $I d\vec{l} \to -I d\vec{l}$ but $\vec{r} \to \vec{r}$, so the fact that $\vec{B}$ is a pseudovector doesn't break the argument. $(\vec{B} \propto I d\vec{l} \times \vec{r})$