

Supplement: Recent questions regarding vectors = pseudovectors
(or polar vectors + axial vectors)

Quick reminder: The nature of a vector is determined by how it reacts to parity operations, or inversions. In a coordinate inversion (or parity transformation), each coordinate transforms into its negative:

$$x \rightarrow -x, \quad y \rightarrow -y, \quad z \rightarrow -z$$

So your typical position vector goes: $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \rightarrow$

$$-x\hat{i} - y\hat{j} - z\hat{k} = -\vec{r}$$

A normal vector, or a polar vector, is one such that $\vec{v} \rightarrow -\vec{v}$ under inversion. Position, velocity, and acceleration all fall under this category.

But consider certain vectors defined in terms of other vectors, like angular momentum

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$$

Under parity, $\vec{r} \rightarrow -\vec{r}$, $\vec{v} \rightarrow -\vec{v}$, so $\vec{L} \rightarrow -\vec{r} \times -\vec{p} = \vec{L}$

\vec{L} doesn't change sign under parity. It's a pseudovector, or axial vector. And that one little difference in transformation behavior turns out to be fantastically important in particle physics, among other places.

Question I (as I recall it)

Does the form of the Lorentz force law come from the fact that \vec{B} is a pseudovector?

Well, since $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} = m\vec{a}$ and \vec{a} is a vector, so must be \vec{F} , \vec{E} , and $\vec{v} \times \vec{B}$.

(Much as you only add scalars to scalars and vectors to other vectors, you should only add pseudovectors to pseudovectors. You can't add a vector to a pseudovector unless you give them a couple of drinks first)

So if $\vec{v} \times \vec{B}$ is a vector, under inversion it must transform like

$$\vec{v} \times \vec{B} \rightarrow -(\vec{v} \times \vec{B})$$

And \vec{v} transforms to $-\vec{v}$, so for all this to work out it must be that $\vec{B} \rightarrow \vec{B}$ and is a pseudovector.

(more \rightarrow)

So let me reframe the question a bit: Is it the fact that \vec{B} is a pseudovector that forces the structure of $\nabla \times \vec{B}$, or is it the fact that magnetic force goes like $\nabla \times \vec{B}$ that forces us to conclude that \vec{B} is a pseudovector? I favor the latter interpretation, since as we'll discover later, things like $\nabla \times \vec{B}$ appear naturally when you go to make the behavior of \vec{E} consistent with relativity.

A field theorist like Dr. Flourenoy might have a more fundamental point of view that changes his answer.

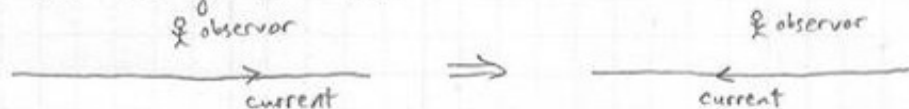
Question II

When making those Ampere's Law symmetry arguments about flipping over a solenoid, have we disrespected the fact that \vec{B} is a pseudovector?

Let's use a straight wire instead since it's a similar case physically but much easier to diagram. My claim was that it can't make a radial field because flipping the wire around wouldn't convert \hat{r} fields to $-\hat{r}$, but according to the Biot-Savart law changing the sign of $I d\vec{l}$ (an equivalent operation) would, so we have a contradiction and the \hat{r} term can't exist.

The counterargument is that under an inversion, \vec{B} should keep the same sign since it's a pseudovector, so there's no problem and we can't rule out the \hat{r} component.

Having pondered this some more I'm fairly convinced that the resolution comes from the wire flipping not being an inversion. When we rotate the wire 180° we go like this:



But in an inversion everything changes sign, including the \hat{r} that represents where the observer is, so a proper inversion would look like:



Put another way, in an inversion we'd change $I d\vec{l} \rightarrow -I d\vec{l}$ and $(\vec{B} \propto I d\vec{l} \times \hat{r})$ $\hat{r} \rightarrow -\hat{r}$ and expect from the Biot-Savart law for the field to stay the same, consistent with \vec{B} 's pseudovector nature. But in the 180° rotation we're performing, $I d\vec{l} \rightarrow -I d\vec{l}$ but $\hat{r} \rightarrow \hat{r}$, so the fact that \vec{B} is a pseudovector doesn't break the argument.