Recap of lecture 2

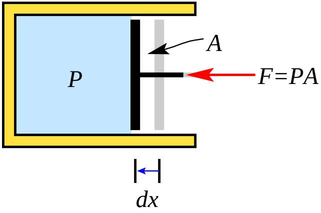
- Equipartition theorem: For classical systems, each quadratic degree of freedom (coordinate or momentum appearing in the Hamiltonian), contributes $\frac{1}{2}kT$ to the total energy. This can be very handy for calculating energies.
- Common degrees of freedom:
  - Translation: one per independent direction
  - Vibration: two per harmonic oscillator mode
  - Rotation: one per principal axis of inertia
- Real systems often exhibit quantum freeze-out of some classical degrees of freedom—$\frac{1}{2}kT$ may be too little energy to lift that part of the system out of its ground state.
- In many common diatomic gases, only 5 (3 translational and 2 rotational) of 8 possible degrees of freedom are active at room temperature, while the two vibrational degrees of freedom may become active at high temperature.

Recap of lecture 2

- We can now allow ideal gases to have nontranslational degrees of freedom.
- First law of thermodynamics: $\Delta U = Q + W$.
- Transfer of heat (Q) and work (W) are two ways of changing the energy of a system. But only the energy itself, not whether it arose from heat or work, is a variable that characterizes the state of the system. That is, energy is a state variable, while heat and work are not.

Compression

A classic example: suppose we compress a gas *quasistatically* with a piston:



The work done on the gas by the applied force during a finite compression (or expansion) is

$$W = -\int_{V_i}^{V_f} P(V)\,dV\,.$$

Compression—an example

[Schroeder, problem 1.31] Imagine some helium in a cylinder with an initial volume of 1 liter and an initial pressure of 1 atm. Somehow the helium is made to expand to a final volume of 3 liters, in such a way that its pressure rises in direct proportion to its volume.

(a) Sketch a graph of pressure vs. volume for this process. (You tell me what it looks like.)

(b) Calculate the work done on the gas during this process, assuming that there are no "other" types of work being done. Since the pressure is proportional to the volume, let's write it as

$$P = \alpha V \,,$$

where α is the proportionality factor, 1 atm/l.

Compression—an example

Since P changes as V does, we have to do the integral:

$$\begin{aligned} W &= -\int_{V_i}^{V_f} P(V)\, dV \\ &= -\alpha \int_{V_i}^{V_f} V\, dV \\ &= -\frac{\alpha}{2}\left(V_f^2 - V_i^2\right) \\ &= -\frac{1\ \mathrm{atm/l}}{2}\left[(3\ \mathrm{l})^2 - (1\ \mathrm{l})^2\right] \\ &= -4\ \mathrm{atm\ l} = -4.052 \times 10^2\ \mathrm{J}\,. \end{aligned}$$

Compression—an example

(c) Calculate the change in the helium's energy content during this process.
Approximate the helium as an ideal gas

$$U = \frac{3}{2}NkT \quad \text{and} \quad PV = NkT\,.$$

Then its energy is determined entirely by P and V:

$$U = \frac{3}{2}PV = \frac{3}{2}\alpha V^2\,.$$

The change in energy is then

$$\Delta U = U_f - U_i = \frac{3}{2}\alpha\left(V_f^2 - V_i^2\right) = -3W = 12.156 \times 10^2 \text{ J}\,.$$

Compression—an example

(d) Calculate the amount of heat added to or removed from the helium during this process.
The first law of thermodynamics (conservation of energy) says

$$\Delta U = Q + W \,.$$

So the heat transferred in the process is

$$Q = \Delta U - W = -3W - W = -4W = 16.208 \times 10^{2}\ \mathrm{J}\,.$$

(e) Describe what you might do to *cause* the pressure to rise as the helium expands. (You tell me.)

Homework

HW Problem

Schroeder problem 1.33, p. 23. Be sure to explain your reasoning in obtaining the signs.

HW Problem

Schroeder problem 1.34, p. 23.

Jargon

- **Isothermal**: *At constant temperature.* Usually refers to some process (e.g., compression). Requires the process rate to be slower than the heat transfer rate.
- **Isotherm**: *A curve with constant temperature.* Refers to a curve on a diagram representing equilibrium states as a function of macroscopic variables, such as pressure and volume.
- **Adiabatic**: *Allowing no heat flow.* May refer to walls (thermally insulating) or processes (fast compared to heat transfer rate).
- **Adiabat**: *A curve with no heat flow.* The progression of equilibrium states along the curve involves no heat flow into or out of the system.

Exercise

I'll collect this one. Grading: 0 (missing), 1 (marginal effort), 2 (good effort).

a. For a compression process on an ideal gas, sketch on a pressure *vs* volume diagram the appearance of two isotherms (with different temperatures).

b. Indicate the direction of progression along the curves during compression (expansion would be the opposite).

c. Indicate which curve corresponds to the higher temperature, and explain why you think so.

d. Now imagine an adiabatic compression process on that ideal gas. In which direction does the temperature change? Explain.

e. Sketch on your diagram a plausible adiabat for compression and indicate the direction of the process.

Exercise

f. (Based on Schroeder, problem 1.38) Two identical bubbles of gas form at the bottom of a lake, then rise to the surface. Because the pressure is much lower at the surface than at the bottom, both bubbles expand as they rise. However, bubble A rises adiabatically (rapidly enough that it exchanges no heat with the water). Bubble B rises isothermally (slowly enough to maintain the same temperature as the water, assumed to be the same everywhere.) Sketch on a pressure *vs* volume diagram the adiabat corresponding to bubble A and the isotherm corresponding to bubble B, indicating clearly which bubble is larger at the surface. Be sure to explain your reasoning. (Approximate the gas as ideal.)