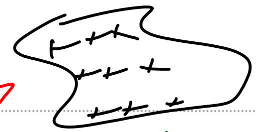
Note Title 6/26/2006

$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$ + other forces



$$\frac{d\vec{P}_{mech}}{\cancel{dt}} = \oint \overleftrightarrow{T}_{EM}\cdot d\vec{a} - \epsilon_0\mu_0 \frac{d}{\cancel{dt}}\int \vec{S}\, d\tau$$

$+\overleftrightarrow{T}_{other}$

momentum in fields with that area

Force/area acting on surface

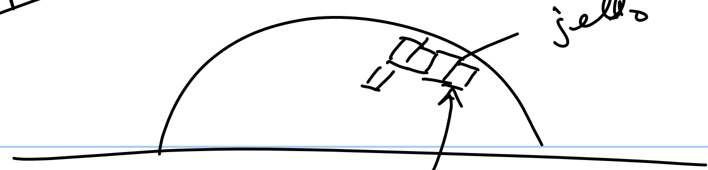
Differential form go densities $\wp$ mom density

$$\vec{\nabla}\cdot\left(-\overleftrightarrow{T}\right) = -\frac{d}{dt}\left(\vec{\wp}_{EM} + \vec{p}_{mech}\right)$$

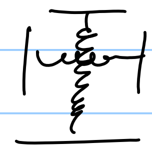
note similarity to $\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$ cons charge

$$\oint \vec{J}\cdot d\vec{a} = -\frac{dQ_{encl}}{dt}$$

Example

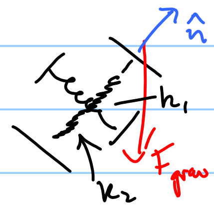


one chunk



Forces: gravity $\quad \overleftrightarrow{T}_{grav}$

spring $\quad \overleftrightarrow{T}_{spring}$



$k_2 \gg k_1$

$$T_{xx}\, da_x + \underbrace{T_{xy}\, da_y + T_{xz}\, da_z}_{\text{shear}}$$

$$d\vec{a} = da\,\hat{n}$$

$$d\vec{a} = da_x\,\hat{x} + da_y\,\hat{y} + da_z\,\hat{z}$$

$$\oint \overleftrightarrow{T} \cdot d\vec{a} = \frac{d\vec{P}}{dt}$$ for static rhs $= 0$

both forces

over one chunk

⊕

$\oint \overleftrightarrow{T}\cdot da = \int_1 \quad \int_2$

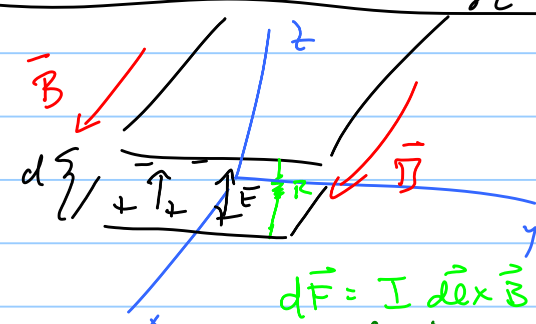
⊕

$\overleftrightarrow{T} = 0$ at ∞

$r = \infty$

Poynting: cons energy $\quad \frac{\partial u_{em}}{\partial t} = -\nabla\cdot\vec{S} \quad$ no mech work

8.6



P

$\mathcal{P}_{em} \quad \frac{\text{mom}}{\text{vol}}$

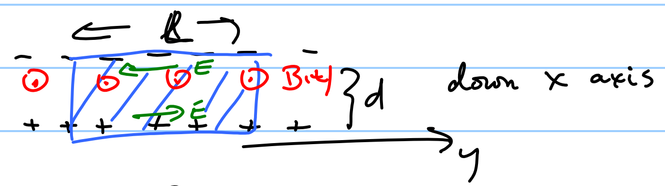
$= \epsilon_0 \vec{E}\times\vec{B}$

$d\vec{F} = I\, d\vec{l}\times\vec{B}$ $\quad$ EM mom

$F = \int I\, d\vec{l}\times\vec{B} = I \int d\vec{l}\times\vec{B} = I\int dz\,\hat{z}\times B_0\hat{x}$

$$\text{Impulse} = \Delta p = \int F\,dt = \int \frac{dp}{\cancel{dt}}\,\cancel{dt} = \Delta p$$

$$I(t) = \frac{dQ}{dt}$$



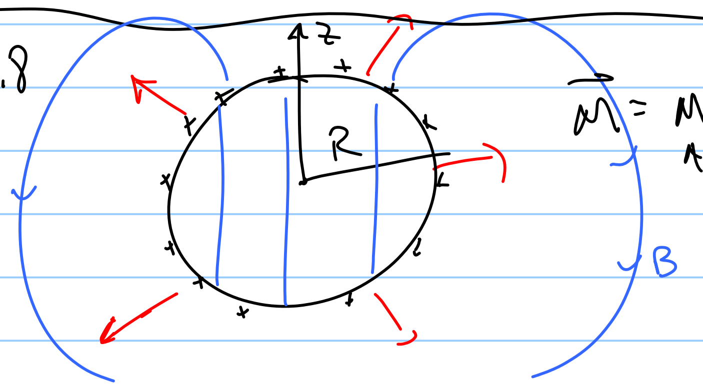
$$\mathcal{E}mf = -\frac{d\Phi_m}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{a} = -\frac{d}{dt}(B\ell d) = \ell d\frac{dB}{dt}$$

$$\oint \vec{E}\cdot d\vec{\ell} = \int_{\text{top}} \vec{E}\cdot d\vec{\ell} + \cancel{\int_{\text{side}} \vec{E}\cdot d\vec{\ell}} + \int_{\text{bottom}} \vec{E}\cdot d\vec{\ell}$$

$$\underbrace{}_{\text{side}=0}$$

$$F = \int dF = \int dq\,E \qquad \text{Impulse} \int F\,dt = \Delta p$$

8.8



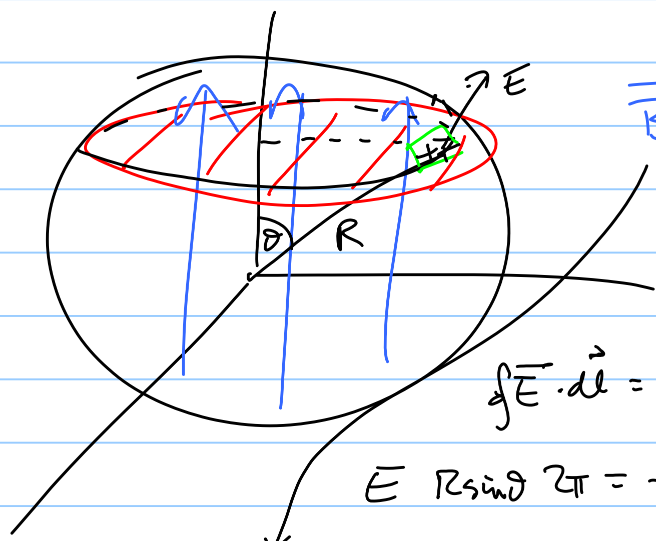
$$\vec{M} = M\hat{z}$$

$\uparrow$ $\dfrac{\text{dipole mom}}{\text{vol}}$

$\vec{\wp}_{em}$ linear mom density $= \epsilon_0 \vec{E}\times\vec{B}$ $\qquad$ $\vec{B}$ Ex. 6.1

$\vec{\ell}_{em}$ ang. mom density $= \vec{r}\times\vec{\wp}_{em}$

$$\int_R^\infty\int_0^\pi\int_0^{2\pi} \ell_{em}\, r^2 \sin\theta\, d\theta\, d\phi = ANS$$



$\vec{B}_{in} = \frac{2}{3}\mu_0 M \hat{z}$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$E\, R\sin\theta\, 2\pi = -\frac{d}{dt}\int \vec{B}\cdot d\vec{a}$$

$$E\, 2\cancel{\pi}\cancel{R}\cancel{\sin\theta} = -\frac{d}{dt} B\pi(R\sin\theta)^2 = \cancel{\pi}R^{\cancel{2}}\sin^{\cancel{2}}\theta\, \frac{2}{3}\mu_0 \frac{dM}{dt}$$

patch on surface of area da $d\vec{F} = \sigma\, da\, \vec{E}$

$\vec{N}$ = Torque $= \vec{r} \times \vec{F} = \dfrac{d\vec{L}}{dt}$

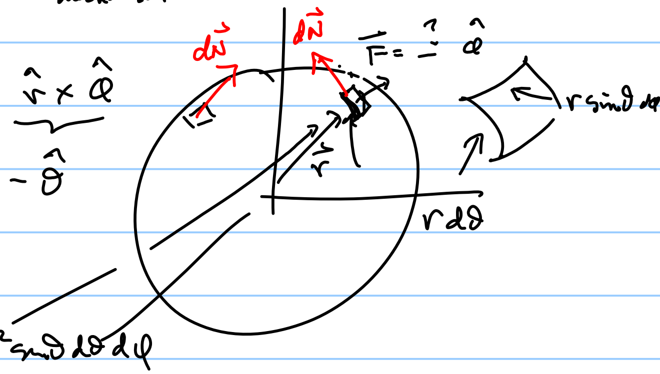
$\vec{F} = \dfrac{d\vec{p}}{dt}$

$\int \vec{F}\, dt = \int \dfrac{d\vec{p}}{dt}\, dt = \Delta p$

$\int \vec{r} \times \vec{F}\, dt = \int d\vec{L} = \Delta \vec{L}_{\text{mechanical}}$

$d\vec{N}_{\text{patch}} = \vec{r} \times d\vec{F} = \underline{\quad ? \quad}\ \underbrace{\hat{r} \times \hat{\varphi}}_{-\hat{\theta}}$

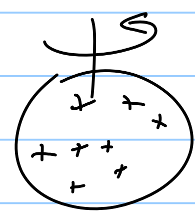
$\qquad\quad =$



Need only $\hat{\theta}_z = -\sin\theta$

$\vec{N} = \int d\vec{N}$ using $da = r^2 \sin\theta\, d\theta\, d\varphi$

8.11 model electron



(a) energy in fields

(b) ang. mom (same as in 8.8(a) with $Q \to e$

$m \to \frac{1}{3} e \omega R^2$

$M \frac{\text{dipole mom}}{\text{vol}}$

$M\, \frac{4}{3}\pi R^3$ = total dipole mom

(a) $r < R$ $\quad E = 0$ $\quad \vec{B} = \frac{2}{3}\mu_0 \sigma R \omega \hat{z}$

$\sigma = \frac{e}{4\pi R^2}$

$r > R$ $\quad E = \frac{1}{4\pi\epsilon_0} \frac{e}{r^2} \hat{r}$

$$\vec{B} = \frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$$

$m = \frac{4}{3}\pi\sigma\omega R^4$

eqn. 5.68 & 5.36