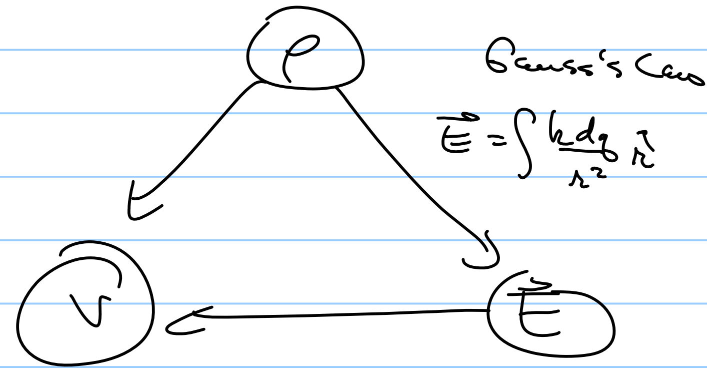
Review Exam I

Note Title 1/28/2009

- find V, $\vec{E}$, & ρ

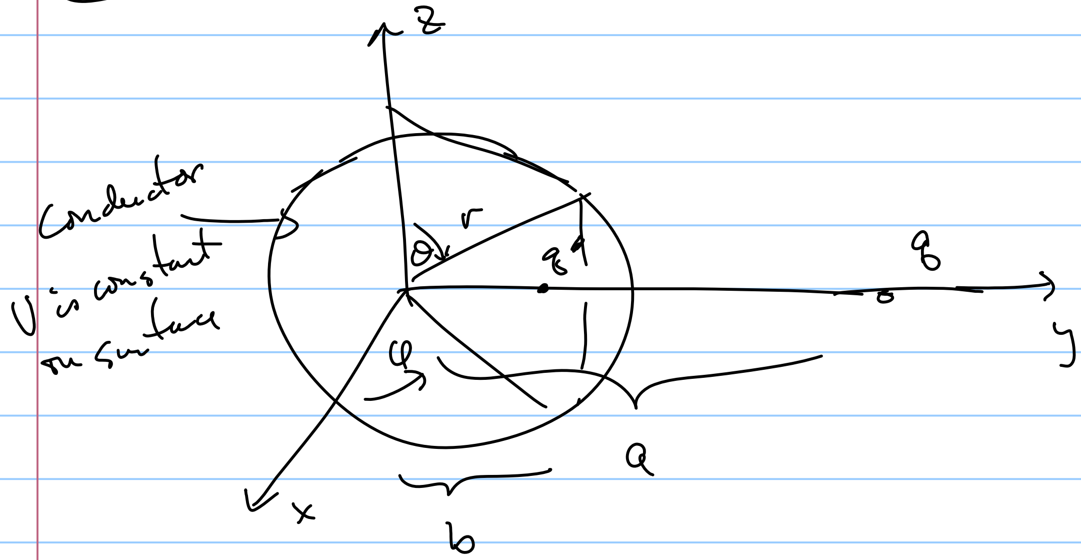


- How to apply V & $\vec{E}$
  (1) Newtons Laws
  (2) Cons. energy

- conductors: $\vec{E} = 0$ inside $\quad \vec{E}_{\parallel} = 0$ surface

- Cal. work required to assemble charge dist.

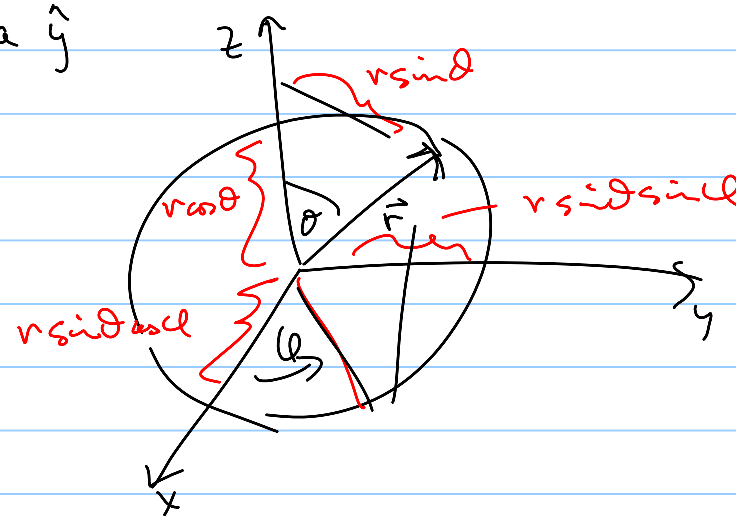
- method images: solves P.D.E. $\nabla^2 V = -\rho/\epsilon_0$
  using bndry condition
  (Gauss's Law used to find σ on surface)



$$V = \frac{kq}{r} \qquad V_{tot} = \frac{kq'}{r_b} + \frac{kq}{r_a} \qquad \vec{r} = \vec{r} - \vec{r}\,'$$

field pt, source pt

$$\vec{r}_b\,' = b\,\hat{y} \qquad \vec{r}_a\,' = a\,\hat{y}$$



$$\vec{r} = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$$

$$x^2 + y^2 + z^2 = r^2$$

$$\vec{r} = r\,\hat{r}$$

$$= r\sin\theta\cos\varphi\,\hat{x} + r\sin\theta\sin\varphi\,\hat{y} + r\cos\theta\,\hat{z}$$

$$\vec{r}_a = r\sin\theta\cos\varphi\,\hat{x} + (r\sin\theta\sin\varphi - a)\,\hat{y} + r\cos\theta\,\hat{z}$$

$$|\vec{r}_a| = \left[\underbrace{r^2\sin^2\theta\cos^2\varphi + r^2\sin^2\theta\sin^2\varphi}_{r^2\sin^2\theta} + a^2 - 2ar\sin\theta\sin\varphi + r^2\cos^2\theta\right]^{1/2}$$

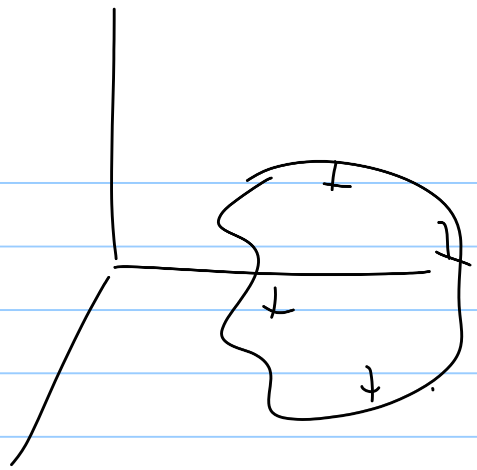
$$r_a = \left[r^2 + a^2 - 2ar\sin\theta\sin\varphi\right]^{1/2}$$

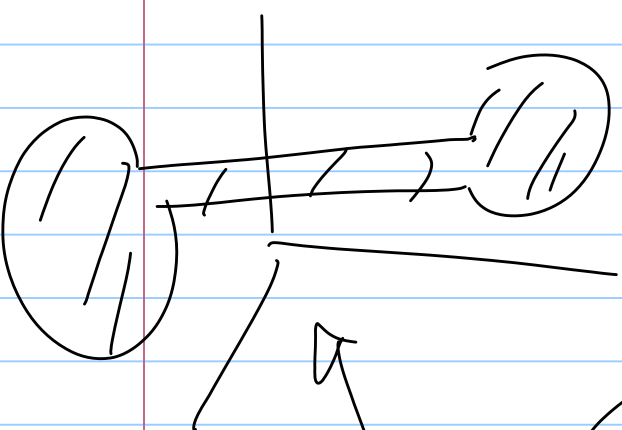
$$V_{tot} = \text{constant} = \boxed{\frac{kq'}{\left[r^2 + b^2 - 2br\sin\theta\sin\varphi\right]^{1/2}} + \frac{kq}{\left[\quad\right]^{1/2}}}$$

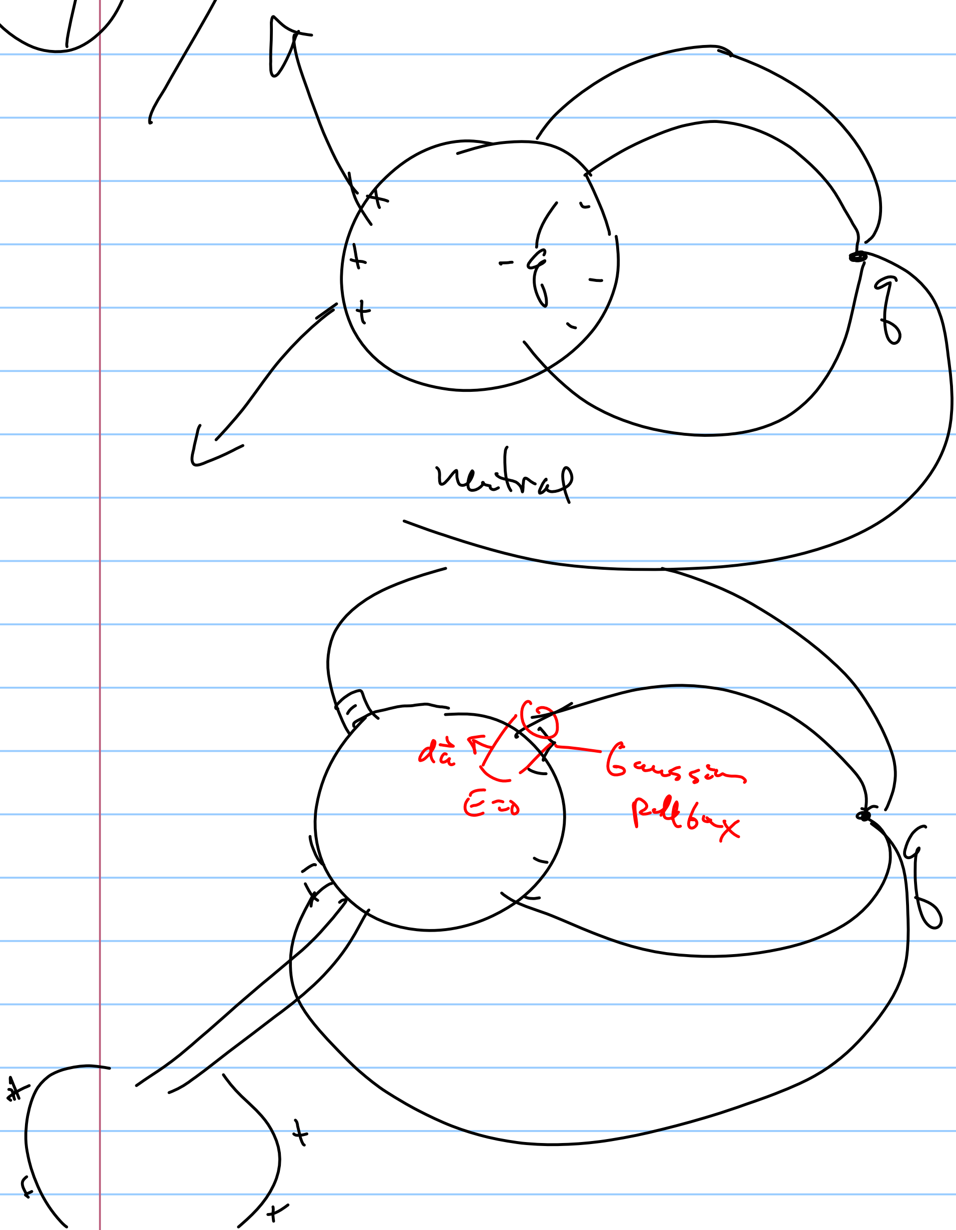
Need constant $= 0$ & $q' = -\frac{R}{a}\,q$

$\int \frac{kq}{r}$ ρ V $\vec{E}$

need $V = 0$ at ∞







$EA = \frac{\sigma A}{\epsilon_0}$

$\sigma = \epsilon_0 E$

$\sigma = \epsilon_0 (-\vec{\nabla} V)$

$$\vec{\nabla} V = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \varphi}\hat{\varphi}$$

Want $\bar{E}_\perp$ near the surface $r \simeq R$

$\bar{E} = -\vec{\nabla} V$ $\quad$ $E_\perp$ is in $\hat{r}$ direction

$E_\perp = \frac{\partial V}{\partial r}$ $\quad$ $\sigma = \epsilon_0 \frac{\partial V}{\partial r}\Big|_{r=R}$

$$Q = \iint \sigma \, da$$