Day 8: Cylindrical Symmetry & Wrap Up / Review

Much of 4.4 is kind of junk (handwavy "proofs", esoteric & poorly explained examples).

The real take-home point is that much of what you know about solving problems with spheres transfers readily to cylinders. For example:

Grounded cylinder in external field. Z axis is axis of cylinder
No z dependence

Let's let $\vec{E}$ lie in the $\phi=0$ direction

For large r, $V=-E_0 r\cos\phi$

Now we solve $\nabla^2 V=0$ in cylindrical, sans z.
We take some basic building blocks, including cylindrical harmonics, & work with

$$V(r,\phi)=A\ln(r)+B+\frac{C\cos\phi}{r}+Dr\cos\phi$$

B is arbitrary, so $B=0$. A is probably zero (we likely need ϕ-dependence).

$D=-E_0$ Can we make $V(a,\phi)=0$ with

$$-E_0 a\cos\phi+\frac{C\cos\phi}{a}\ ?$$

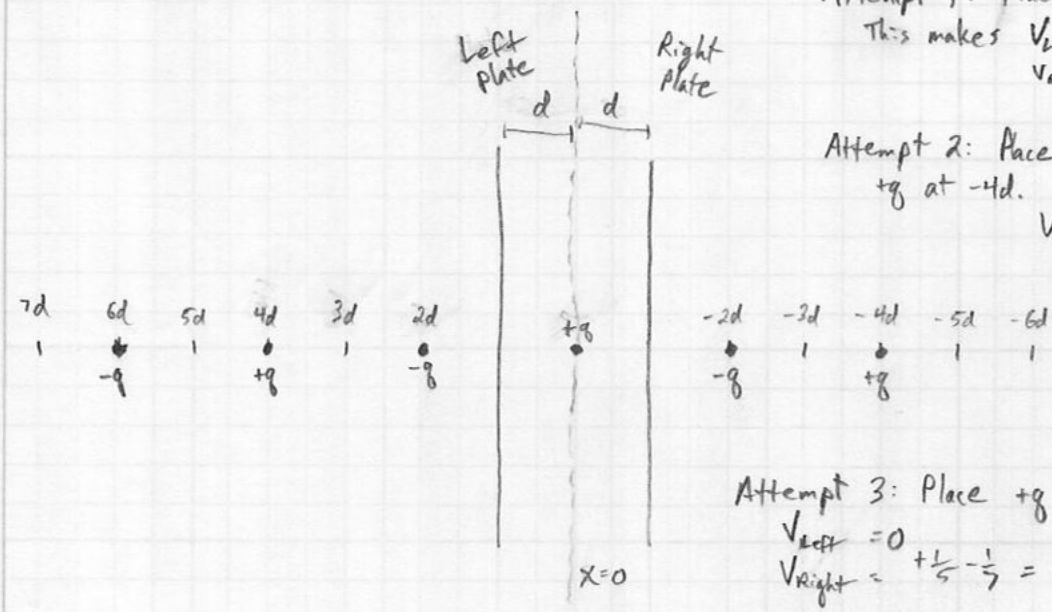
$$\frac{C}{a}=-E_0 a \Rightarrow C=E_0a^2$$

And thus $V(r,\phi)=-E_0 r\cos\phi+\frac{E_0 a^2\cos\phi}{r}$

From there $\vec{E}$ and σ follow.

Line by line, nearly identical to the grounded sphere.
But this is nothing new; spherical & cylindrical problems often differ only slightly (see: Phys 200 Gauss's Law problems).

Close out with another planar problem. Two grounded parallel planes separated by $2d$, with a charge $+q$ in the middle of them. Let's see if we can build an image solution by satisfying the BCs, $V=0$ at each plate.



Attempt 1: Place $-q$ at $2d$.
This makes $V_{Left}=0$,
$V_{Right} = +1 - 1/3 = 2/3$ units

Attempt 2: Place $-q$ at $-2d$ and $+q$ at $-4d$. $V_{Right}=0$
$V_{Left} = 1 - 1 - 1/3 + 1/5 = -\frac{2}{15}$ unit
Closer!

Attempt 3: Place $+q$ at $4d$, $-q$ at $6d$
$V_{Left} = 0$
$V_{Right} = +\frac{1}{5} - \frac{1}{7} = \frac{2}{35}$ units

If we keep doing this, V quickly converges to zero at either plane. We have infinitely many images, and the voltage in the space between the planes at any point on the $x=0$ plane comes from:

images come in pairs

alternate in signs

$$V(r) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r} + \sum_{k=1}^{\infty} \frac{2(-1)^k}{\sqrt{r^2 + (2dk)^2}}\right]$$

Term for real charge, $r = \sqrt{y^2+z^2}$

y^2+z^2

x^2, each pair is $2d$ farther out than the previous.

And from here, we could get σ and $\vec{E}$.