For each problem justify the points you deserve by circling where you satisfied the rubric and putting a circle around your point value. You can give partial credit. Total your points next to your name.

Exam 3 **March 7** **Name**.....Scoring Rubric..........

1. Write Stokes theorem.

$$\underbrace{\int \vec{\nabla}\times\vec{V}\cdot d\vec{a}}_{5\text{ pts}} = \underbrace{\oint \vec{V}\cdot d\vec{r}}_{5\text{ pts}}$$

2. Write the divergence theorem.

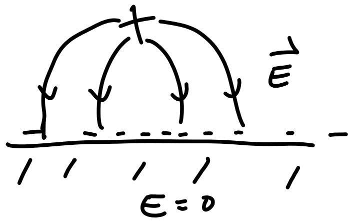
$$\underbrace{\int \vec{\nabla}\cdot\vec{V}\,d\tau}_{5\text{ pts}} = \underbrace{\oint \vec{V}\cdot d\vec{a}}_{5\text{ pts}}$$

3. Write conservation of charge in integral and differential form.

$\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$ 5 pts $\qquad \oint \vec{J}\cdot d\vec{a} = -\frac{\partial}{\partial t}\int \rho\, d\tau$ 5 pts

4. A charge q is brought to a perpendicular distance d from a conductor filling the x-y plane. The voltage above the plane is given by $V = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{\sqrt{x^2+y^2+(z-d)^2}} - \frac{q}{\sqrt{x^2+y^2+(z+d)^2}}\right)$. Write an integral expression for the work required to assemble the charges. Explain the limits and how you calculate all parameters in the integrand

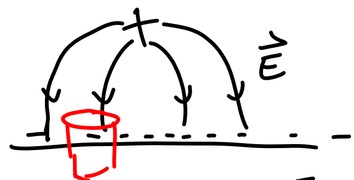
Three possible methods with pts are:



1.) $W = \frac{\epsilon_0}{2}\int E^2\,dx\,dy\,dz$ 5 pts, all space 5 pts, $\vec{E} = -\vec{\nabla}V$ 5 pts

but $E=0$ in conductor so really only in upper half plane

2.) $W = \frac{1}{2}\int \rho V\,d\tau \rightarrow \frac{1}{2}\int \sigma V\,da$ 5 pts, on metal surface 5 pts, 5 pts

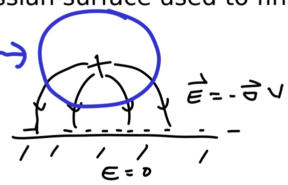


Gaussian surface used to find $E = \frac{\sigma}{\epsilon_0}$

3.) $W = \frac{\epsilon_0}{2}\left(\int E^2\,d\tau + \oint V\vec{E}\cdot d\vec{a}\right)$ v enclosed by surface

5 pts for integrands

5 pts for limits $\qquad$ 5 pts for $\vec{E} = -\vec{\nabla}V$



5. Show why a method of solution to Laplace's equation (like the relaxation method) can be used to solve for the current density in an Ohmic material under steady state conditions.

$\vec{J} = \sigma\vec{E}$ Ohm's law 5 pts

$\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$ conservation of charge

$\frac{\partial\rho}{\partial t} = 0$ in steady state 5 pts

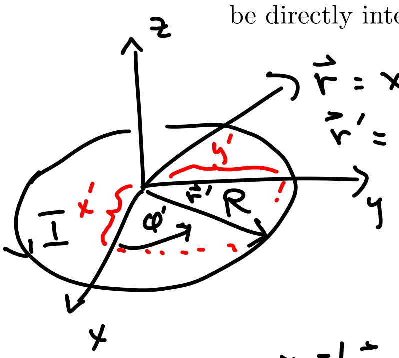
$\vec{\nabla}\cdot(\sigma\vec{E}) = \sigma\vec{\nabla}\cdot\vec{E} = 0$ 5 pts

$\vec{\nabla}\cdot\vec{E} = -\nabla^2 V = 0$ Laplaces eqn 5 pts

$\vec{E} = -\vec{\nabla}V$

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,')\,d\tau'}{\mathscr{r}} \rightarrow \frac{\mu_0}{4\pi}\int \frac{I_0\,d\vec{r}\,'}{\mathscr{r}}$$ 5 pts

6. Write an integral with limits for the vector potential at an arbitrary point from a circular wire of radius R centered in the x-y plane which carries current I_0. Do this in sufficient detail so that it could be directly integrated. Do not integrate!



$$\left.\begin{aligned}\vec{r} &= x\hat{x} + y\hat{y} + z\hat{z}\\ \vec{r}\,' &= x'\hat{x} + y'\hat{y} + 0\hat{z}\\ x' &= R\cos\varphi'\\ y' &= R\sin\varphi'\end{aligned}\right\}$$ 5 pts

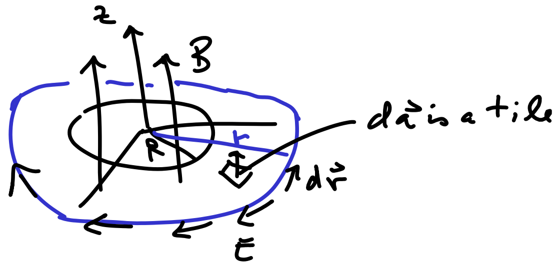
$$\underbrace{d\vec{r}\,' = -R\sin\varphi'\,d\varphi'\,\hat{x} + R\cos\varphi'\,d\varphi'\,\hat{y}}_{5\text{ pts}}$$

$$\mathscr{r} = |\vec{r} - \vec{r}\,'| = \sqrt{(x - R\cos\varphi')^2 + (y - R\sin\varphi')^2 + z^2}\ \Big\}$$ 3 pts

$$\vec{A} = \frac{\mu_0}{4\pi}\int_0^{2\pi} \frac{I_0(-R\sin\varphi' + R\cos\varphi')\,d\varphi'}{\mathscr{r}}$$

limits 2 pts

7. A uniform magnetic field $B(t)\hat{z}$ is increasing with time and just fills a circular region in the x-y plane of radius R centered at the origin. Derive an expression for the induced electric field for $r > R$ and specify its direction. Justify the steps in the derivation.



$$\oint \vec{E}\cdot d\vec{r} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{a}\ \Big\}$$ 5 pts

$$\oint |\vec{E}||d\vec{r}|\cos 0 = E\oint dr = E\,2\pi r\ \Big\}$$ 5 pt LHS

E is the same for all dr's

$$-\frac{d}{dt}\int |\vec{B}|\cdot|d\vec{a}|\cos 0 = -\frac{d}{dt} B\pi R^2$$ since $B \neq 0$ only in circle of radius R } 5 pts RHS

B is the same for all da's

$$E\,2\pi r = -\frac{d}{dt} B\pi R^2$$

$$\vec{E} = -\frac{\pi R^2}{2\pi r}\frac{d}{dt}B\,\hat{\varphi}$$

} 5 pts for final form including direction