ticc.mines.edu/csm/wiki/

Go to Physics → Summer 2010

E&M.

Maxwell's Eqns

$$\vec{V}_1 = \vec{V}_2 \Rightarrow \begin{array}{l} V_{1x} = V_{2x} \\ V_{1y} = V_{2y} \\ V_{1z} = V_{2z} \end{array}$$

---

(1) $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \Rightarrow \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\rho}{\epsilon_0}$

(2) $\vec{\nabla} \cdot \vec{B} = \emptyset \Rightarrow \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = \emptyset$

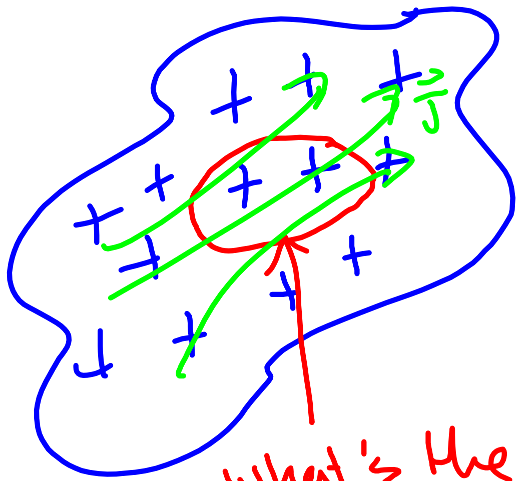
(3) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (Faraday's Law)

(4) $\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$ (Modified Ampere's Law)

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

$\vec{f} = \rho \vec{E} + \vec{J} \times \vec{B}$

↑ Force density.



what's the force on the charge in a volume enclosed by red circle if charge dens. is ρ, current dens. is J, and you have $\vec{E}$, $\vec{B}$.

$\vec{F} = \int \vec{f}\, dV = \int \rho \vec{E} + \vec{J} \times \vec{B}\, dV$

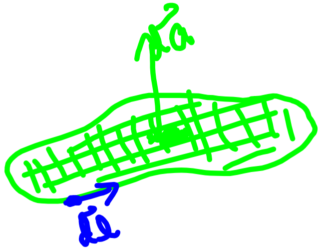
(1) $\int \vec{\nabla}\cdot\vec{E}\,dV = \frac{1}{\epsilon_0}\int \rho\,dV$

$$\oint \vec{E}\cdot d\vec{a} = \frac{1}{\epsilon_0} q_{enc}$$

(2) $\int \vec{\nabla}\cdot\vec{B}\,dV = 0 \Rightarrow \oint \vec{B}\cdot d\vec{a} = 0$

(3) $\int \vec{\nabla}\times\vec{E}\cdot d\vec{a} = \int -\frac{\partial\vec{B}}{\partial t}\cdot d\vec{a}$

$$\int \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{a}$$



(4) $\int \vec{\nabla}\times\vec{B}\cdot d\vec{a} = \int \mu_0\epsilon_0 \frac{\partial\vec{E}}{\partial t}\cdot d\vec{a} + \int \mu_0 \vec{J}\cdot d\vec{a}$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\epsilon_0\frac{\partial}{\partial t}\int \vec{E}\cdot d\vec{a} + \mu_0 I_{enc}$$

Maxwell's equations in matter

Bound charge and bound current.

$-\vec{\nabla}\cdot\vec{P} = \rho_b$ $\qquad$ $\vec{\nabla}\times\vec{M} = \vec{J}_b$

$\vec{D} \equiv \epsilon_0\vec{E} + \vec{P}$ $\qquad$ $\vec{H} \equiv \frac{1}{\mu_0}\vec{B} - \vec{M}$

In a linear material

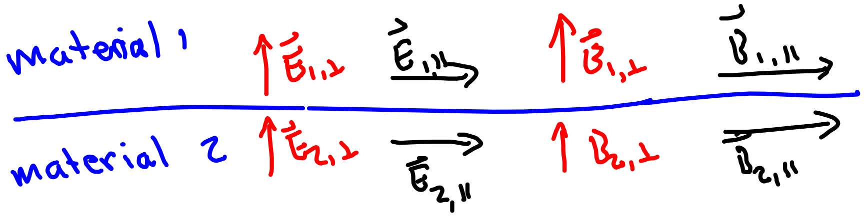
$\vec{P} \propto \vec{E} \Rightarrow \vec{P} = \epsilon_0 \chi_e \vec{E}$; $\vec{M} = \chi_m \vec{H}$

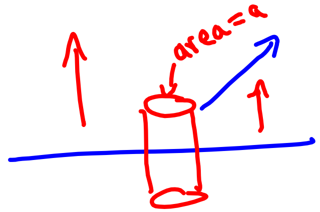
$\vec{D} = \epsilon\vec{E}$ $\qquad$ $\vec{H} = \frac{1}{\mu}\vec{B}$

$\vec{D} = \epsilon_r\epsilon_0\vec{E}$ $\qquad$ $\vec{H} = \frac{1}{\mu_r\mu_0}\vec{B}$

n (index of refraction)

$= \sqrt{\epsilon_r}$





$$\oint \vec{E}\cdot d\vec{a} = \frac{1}{\epsilon_0}\int \rho\, dV$$

$$E_{1,\perp}a - E_{2,\perp}a + \cancel{\int \vec{E}_1\cdot d\vec{a}\Big|_{sides}}^{\emptyset} + \cancel{\int \vec{E}_2\cdot d\vec{a}\Big|_{sides}}^{\emptyset}$$

$$E_{1,\perp}\cancel{a} - E_{2,\perp}\cancel{a} = \frac{1}{\epsilon_0}\int \sigma\, da$$
$$= \frac{\sigma}{\epsilon_0}\cancel{a}$$

$$\boxed{E_{1,\perp} - E_{2\perp} = \frac{\sigma}{\epsilon_0}}$$

$$D_{1,\perp} - D_{2,\perp} = \frac{\sigma_{free}}{\epsilon_0}$$

Linear materials without free charge.

$$\epsilon_1 E_{1,\perp} - \epsilon_2 E_{2,\perp} = \emptyset$$

Summary of BCs

Linear materials:

$\epsilon_1 E_{1\perp} - \epsilon_2 E_{2\perp} = \phi$, $E_1^{\parallel} - E_2^{\parallel} = \phi$

$B_{1\perp} - B_{2\perp} = \phi$, $\frac{1}{\mu_1} B_1^{\parallel} - \frac{1}{\mu_2} B_2^{\parallel} = \phi$