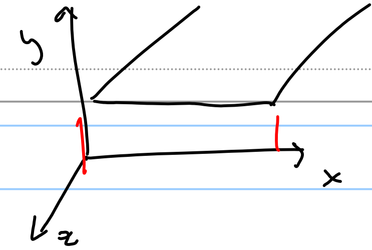
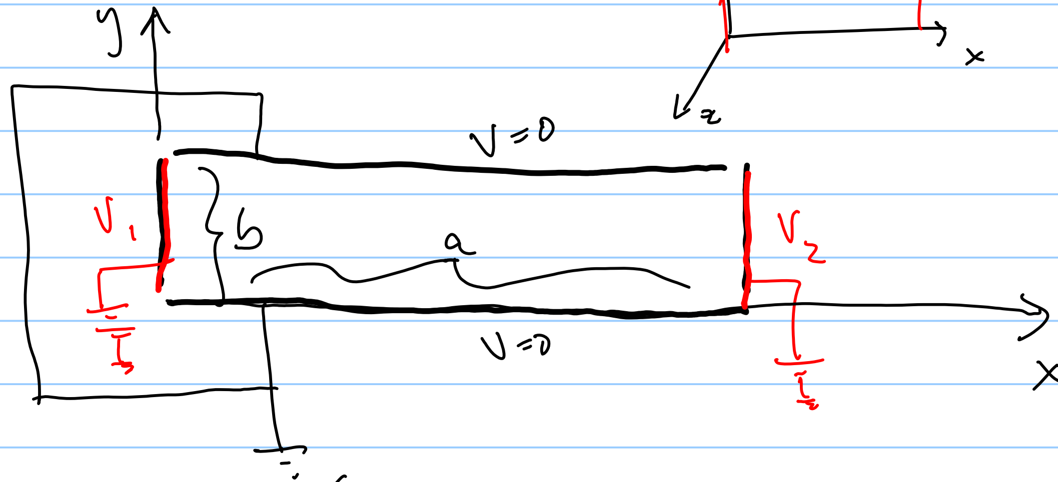
Note Title 2/4/2009

Example Sep. variables





Boundary conditions
$$\begin{cases} x=0 & V=V_1 \\ x=a & V=V_2 \\ y=0,b; & V=0 \end{cases}$$

$$V(x,y,z)=X\,Y\,Z$$

$$\underbrace{\frac{1}{X}\frac{d^2X}{dx^2}}_{C_1} + \underbrace{\frac{1}{Y}\frac{d^2Y}{dy^2}}_{C_2} + \underbrace{\frac{1}{Z}\frac{d^2Z}{dz^2}}_{C_3} = 0$$

$$C_1 + C_2 + \underset{\overset{\shortparallel}{0}}{C_3} = 0$$

$$Z(z) = \text{const} \implies C_3 = \emptyset$$

Satisfy BC's on $Y(y)$ choose $\frac{d^2Y}{dy^2} = \underbrace{-k^2}_{C_2} Y$ where $k^2>0$

$$Y(y) = A\cos ky + B\sin ky$$

$y=0 \quad V=0 \qquad Y(0) = \underset{\overset{\shortparallel}{0}}{A}(1) + B(0) = 0$

$y=b \quad V=0 \qquad Y(b) = B\sin kb = 0 \qquad kb = n\pi$

$n=1,2,3,\ldots$

$$\boxed{k = \frac{n\pi}{b}}$$

$$C_1 - k^2 + 0 = 0$$

$$C_1 = k^2$$

$$\frac{d^2 \overline{X}}{dx^2} = C_1 \overline{X} = k^2 \overline{X}$$

$$\overline{X}(x) = G e^{kx} + H e^{-kx}$$

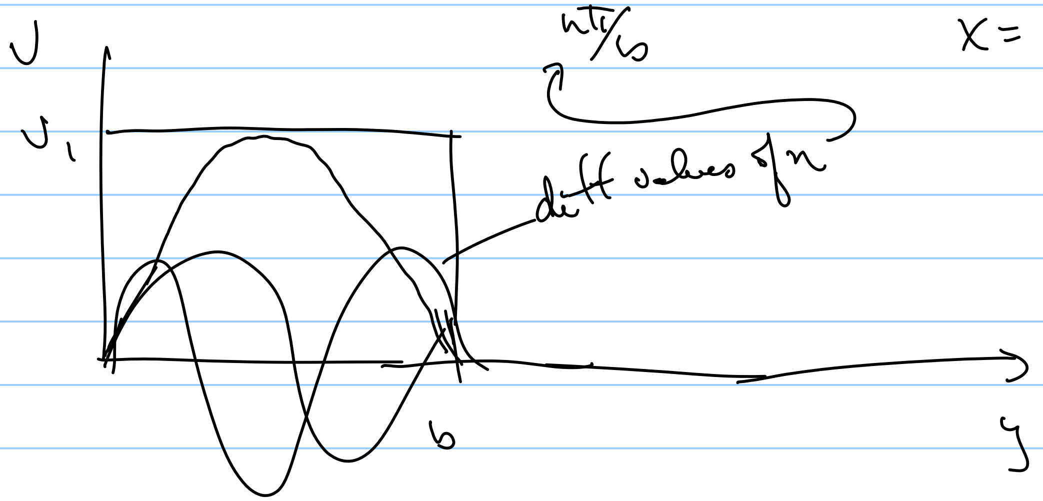
$$V(x,y) = \overline{X}\,\overline{Y} = \left(G e^{kx} + H e^{-kx}\right) \sin ky$$

const A into

bndry on $x = 0$ $\quad V = V_1$

$$V(0,y) = (G + H) \sin \underset{\overset{\parallel}{n\pi/b}}{k} y = V_1 \quad \text{const}$$

$x = 0$



$$V = \sum_i A_i V_i \qquad \text{fourier series}$$

$$V = \sum_{n=1}^{\infty} \left(A_n e^{-\frac{n\pi x}{b}} + B_n e^{\frac{n\pi x}{b}}\right) \sin\left(\frac{n\pi y}{b}\right)$$

Satisfy bndry cond at $x=0$

$$V(0,y) = \sum_1' (A_n + B_n) \sin\frac{n\pi y}{b}$$

$\Downarrow V_1$

Find A_n & B_n & then I'm done!

multiply both sides of eqn $\sin\left(\frac{m\pi y}{b}\right)$

$$\int_0^b V_1 \sin\left(\frac{m\pi y}{b}\right) dy = \int_0^b \sum_1' (A_n + B_n) \sin\left(\frac{n\pi y}{b}\right) \sin\frac{m\pi y}{b}\, dy$$

mathematica m, n elements integers to do integral

$$\underbrace{\int_0^b \sin\left(\frac{n\pi y}{b}\right) \sin\left(\frac{m\pi y}{b}\right) dy}_{\text{sym} \quad \text{anti}} = \frac{b}{2}\,\delta_{nm}$$

0 if $n \neq m$
1 if $n = m$

1-D wave function in Quantum expanded in terms of sep. variables solns.

$$\psi(x) = \sum_n' a_n \psi_n(x)$$

hydrogen atom wavefunction

bndry condition orbiting electron need to find a_n

$$\int \psi(x)\, \psi_m(x)\, dx = \sum a_n \underbrace{\int \psi_n(x)\, \psi_m(x)\, dx}_{\delta_{nm}}$$

$x=0$ initial condition

even & odd function integrated over a symmetric interval help to integral