4_23_08

Note Title 4/23/2008

Classical : current loops

$d\vec{S}$ oriented area element

$$d\vec{\mu} = I \, d\vec{S}$$

magnetic moment

in Atoms $\vec{\mu}$ is associated with orbiting electrons

$$\vec{\mu} = \gamma \vec{L}$$

gyromagnetic ratio

Placed in a magnetic field $\vec{B}$ there is an energy associated w/ the mag. moment

$$E = -\vec{\mu} \cdot \vec{B}$$

energy is minimized when $\vec{\mu}$ lies along $\vec{B}$.

There is also a torque

$$\vec{G} = \vec{\mu} \times \vec{B}$$

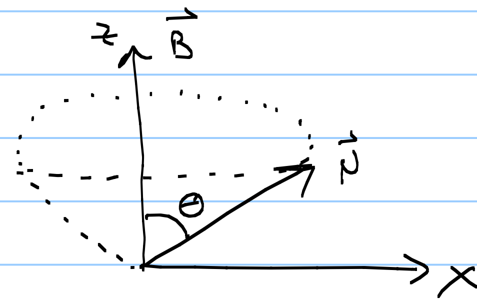
Since $\vec{\mu}$ is associated with ang. mom, and since torque $= \frac{d}{dt}$ angular mom.

$$\frac{d}{dt}\vec{L} = \frac{1}{\gamma}\frac{d}{dt}\vec{\mu} = \vec{\mu}\times\vec{B}$$

$$\boxed{\frac{d\vec{\mu}}{dt} = \gamma\,\vec{\mu}\times B}$$

so change in $\vec{\mu}$ $\perp$ Both $\vec{\mu}$ + $\vec{B}$.

E.g



so $\qquad \dot{\mu}_x = \gamma B \mu_y$

$$\dot{\mu}_y = -\gamma B \mu_x$$

$$\dot{\mu}_z = 0 \Rightarrow \mu_z = |\vec{\mu}|\cos\theta$$

$$\mu_x = |\vec{\mu}|\sin\theta\sin(\omega_L t)$$

$$\mu_y = |\vec{\mu}|\sin\theta\cos(\omega_L t)$$

$\boxed{\omega_L = \gamma B}$ Larmour freq.

spinning top.

idea of current loops can be extended to the magnetic moments of

orbital electrons (ECR, e^- cyclotron res)
electron spins (ESR, e^- spin resonance)
nuclear spins (NMR, nucl. mag. reson)

each with their own gyromagnetic ratio

E.g. for proton spin, at $B = 19$ Tesla, $\omega_L \approx 800$ MHz.

most fundamental QM spin idea: quantization of spin in z directions. Stern-Gerlach exp.

new applications
- spintronics
- spin polarization
- Quantum Computing (qubits)
- Plasma Heating

Back at the ranch

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

char. polynomial: $\text{Det}\begin{pmatrix} -\lambda & \hbar/2 \\ \hbar/2 & -\lambda \end{pmatrix} = 0$

$$\Rightarrow \lambda^2 - \left(\frac{\hbar}{2}\right)^2 = 0$$

$\lambda = \pm \frac{\hbar}{2}$ Same as for S_z

$$\chi_+^{(x)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \chi_-^{(x)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$\Downarrow$ w/ eigenvalue $\frac{\hbar}{2}$ $\qquad$ $\Downarrow$ w/ eigenvalue $-\frac{\hbar}{2}$

$$S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\text{Det}\begin{vmatrix} -\lambda & -i\hbar/2 \\ i\hbar/2 & -\lambda \end{vmatrix} = \lambda^2 - \left(\frac{\hbar}{2}\right)^2 = 0$$

$$\Rightarrow \lambda = \pm \hbar/2$$

eigenvectors

$$\chi_+^{(y)} = \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \end{pmatrix}, \qquad \chi_-^{(y)} = \frac{1}{\sqrt{2}}\begin{pmatrix} -i \\ 1 \end{pmatrix}$$

$$\hat{S}^2 |\psi\rangle = (\hat{S}_x^2 + \hat{S}_y^2 + \hat{S}_z)|\psi\rangle$$

$$= \left(\left(\frac{\hbar}{2}\right)^2 + \left(\frac{\hbar}{2}\right)^2 + \left(\frac{\hbar}{2}\right)^2 \right) |\psi\rangle$$

$$= \frac{3\hbar^2}{4} |\psi\rangle$$

which we know as $s(s+1)\hbar^2$

$$[S_x, S_y] = i S_z$$
$$[S^2, S_z] = 0$$

This says it **is** simultaneously possible to know the total spin and one of its components

It **is not** possible to know more than one component simultaneously

in general

$$[\hat{\vec{S}} \cdot \vec{x}, \hat{\vec{S}}] = i\, \hat{\vec{S}} \times \vec{x}$$

where $\vec{x}$ points in any direction

show that:

$$\hat{S}_{\theta,\varphi} = \sin\theta\cos\varphi\,\hat{S}_x + \sin\theta\sin\varphi\,\hat{S}_y + \cos\theta\,\hat{S}_z$$

the spin operator for spin component along direction determined by polar angles θ, φ, has eigenvalues $\pm\hbar/2$

and eigenvectors

$$\chi_+^{\theta,\varphi} = \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2)\, e^{i\varphi} \end{pmatrix}$$

$$\chi_-^{\theta,\varphi} = \begin{pmatrix} \sin(\theta/2) \\ -\cos(\theta/2)\, e^{i\varphi} \end{pmatrix}$$

Generic spinor can be represented in terms of eigenvectors of S_z

$$\chi = a\,\chi_+ + b\,\chi_-$$
$$= a\begin{pmatrix}1\\0\end{pmatrix} + b\begin{pmatrix}0\\1\end{pmatrix}$$

so $$S_z\,\chi = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} a\begin{pmatrix}1\\0\end{pmatrix} + \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} b\begin{pmatrix}0\\1\end{pmatrix}$$

$$= a\frac{\hbar}{2}\begin{pmatrix}1\\0\end{pmatrix} - b\frac{\hbar}{2}\begin{pmatrix}0\\1\end{pmatrix}$$

So $\quad S_z \chi = a\chi_+^{(z)} + b\chi_-^{(z)}$

arbitrary spinor (spin state)

This is the spin part of the electrons wavefcn i.e. its a 2-comp vector rather than a function

$\Rightarrow$ prob. of measuring $+\frac{\hbar}{2}$ is $|a|^2$

prob. of measuring $+\frac{\hbar}{2}$ is $|b|^2$

E.g. Suppose $\chi = \frac{1}{\sqrt{6}}\begin{pmatrix} 1+i \\ 2 \end{pmatrix}$

NB $\quad \chi^2 = \frac{1}{6}\left((1+i)(1-i)+4\right)$

$= \frac{1}{6}(1+1+4) = 1$

problem what is the probability of measuring $+\frac{\hbar}{2}$ in an S_z measurement. Answer first write χ as a superposition of S_z eigenstates

$$\chi = \frac{1}{\sqrt{6}}\begin{pmatrix} 1+i \\ 2 \end{pmatrix} = a\begin{pmatrix} 1 \\ 0 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$\Rightarrow \quad a = \frac{1}{\sqrt{6}}(1+i) \qquad b = \frac{1}{\sqrt{6}}2$

So the answer is $|a|^2 = \frac{2}{6} = \frac{1}{3}$.

Now suppose we want to measure S_x. Now we want to re-express in terms of S_x eigenstates:

$$\frac{1}{\sqrt{6}}\begin{pmatrix} 1+i \\ 2 \end{pmatrix} = a\,\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + b\,\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\Rightarrow \quad 1+i = a\sqrt{3} + b\sqrt{3}$$
$$2 = a\sqrt{3} - b\sqrt{3}$$

$$\Rightarrow \quad 3+i = 2\sqrt{3}\,a \quad \Rightarrow a = \frac{3+i}{2\sqrt{3}}$$

and $i-1 = 2\sqrt{3}\,b \qquad b = \dfrac{i-1}{2\sqrt{3}}$

So prob. of measuring $+\frac{\hbar}{2}$ is

$$|a|^2 = \frac{(3+i)(3-i)}{12} = \frac{9+1}{12} = \frac{5}{6}$$

$|b|^2$ must be $1-\frac{5}{6}$. check:

$$|b|^2 = \frac{(i-1)(i+1)}{12} = \frac{2}{12} = \frac{1}{6} \quad \checkmark$$